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INERTIA AND CONVECTION EFFECTS IN HYDRODYNAMIC LUBRICATION FOR A SLIDER BEARING

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "INERTIA AND CONVECTION EFFECTS IN HYDRODYNAMIC LUBRICATION FOR A SLIDER BEARING", submitted by MOHAMMAD IQBAL ANWAR in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

The contribution of inertia terms towards load bearing capacity, the effect of convection terms on the temperature distribution and the temperature variation in high and low viscosity lubricants, are considered for a plane slider bearing. The fluid was to be assumed incompressible of constant but arbitary viscosity. Through a stream function approach and a power series expansion, the momentum equation is converted into a set of ordinary differential equations which are numerically solved. The pressure distribution, load capacity, shear stress and the increase in load capacity is calculated for various modified Reynolds numbers, $\overset{*}{Re}$. The energy equation is also solved numerically with and without convective terms. The temperature distribution is obtained. It was found that the presence of the inertia terms increases the load capacity. This effect becomes important at high $\overset{*}{Re}$. The analysis indicates that the convection terms can not be neglected and that the lubricating oils of high viscosity develop larger temperature variation in the film than those of low viscosity.

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LIST OF SYMBOLS

a = coefficient in difference equation (A.5), (B.5) and (B.14)

A = coefficient in difference equation (A.1)

b = coefficient in difference equation (A.5), (B.5) and (B.14)

B = coefficient in difference equation (A.1)

C_0 = a constant in expression (2.28)

C_1 = a constant in expression (2.28)

C_2 = a constant in expression (2.28)

C_3 = a constant in expression (2.28)

C = coefficient in difference equation (A.5), (B.5) and (B.14)

C_p = specific heat at constant pressure

d = coefficient in difference equation (A.5), (B.5) and (B.14)

D = a constant in recurrance relation (3.1)

e = coefficient in difference equation (A.5)

E = type of Eckert number, U^2/C_pT_s

f = function related to the stream function by

$$\psi = K \sum_{n=0}^{\infty} f_n (\eta) \delta^n$$

G = C_1/K

h = film thickness h(x)

h_i = film thickness at inlet

h_m = mean film thickness

h_0 = film thickness at outlet

h_r = film ratio, h_i/h_o

H	=	step magnitude in y-direction
K	=	ν/h'
l	=	step magnitude in x-direction
L	=	length of bearing
m	=	an integer in numerical method
m	=	total mass flow rate per unit width
n	=	an integer in numerical method
N	=	an integer in numerical method
p	=	gauge pressure
$\overline{p}$	=	dimensionless pressure
P	=	Prandtl number, $C_p\mu/\kappa$
$[P]$	=	field matrix of p
R	=	a constant in Recurrance Relation (3.1)
R	=	radius of the runner
Re^*	=	modified Reynolds number, $UL/\nu\ (h_0/L)^2$
T	=	temperature of lubricant
T_s	=	lubricant supply temperature
$\overline{T}$	=	dimensionless temperature
u	=	velocity in x-direction
$\overline{u}$	=	dimensionless velocity in x-direction
U	=	velocity of moving component
$[U]$	=	field matrix of u
v	=	velocity in y-direction

$\bar{v}$ = dimensionless velocity in y-direction

V = reference velocity in y-direction

$[V]$ = field matrix of v

S = dummy variable used in numerical method

W = load capacity

$\bar{W}$ = dimensionless load capacity

$\bar{W}_L$ = dimensionless load capacity

x = longitudinal co-ordinate

$\bar{x}$ = dimensionless longitudinal co-ordinate

y = transverse co-ordinate

$\bar{y}$ = dimensionless transverse co-ordinate

Subscript

0 condition at moving component

Superscripts

$'$ differentiation with respect to independent variable

Greek Letters

δ = h/h_0

η = y/h

κ = thermal conductivity of lubricant

μ = dynamic viscosity

LIST OF SYMBOLS (continued)

ν	= kinematic viscosity
ρ	= density
τ	= shear stress
$\bar{\tau}$	= dimensionless shear stress
ψ	= stream function
ω	= angular velocity

CHAPTER I

STATEMENT OF THE PROBLEM AND REVIEW OF RELEVANT LITERATURE

1.1 INTRODUCTION

The classical theory of hydrodynamic lubrication is based upon the assumption that the dominating influence is that of lubricant viscosity. In the narrow space between two inclined surfaces in relative motion, the pressure generated in a fluid film by viscous forces tend to keep the surfaces apart, thus forming a bearing. The most favourable combinations of geometry and kinematics of such system have been much sought after. Predictions are usually required of the main dependent variables pressure, load capacity, shear stress, mass flow rate and temperature distribution; these are influenced by the geometry of the system, the kinematics of the system and the properties of the lubricant.

Specifically, the basic differential equations for the study of hydrodynamic lubrication are the continuity equation, the momentum equations, the energy equations and the equation of state. These equations, interdependent as they are through the physical properties of the lubricant, are extremely complicated mathematically. Several simplifying assumptions have to be made before a solution could be obtained.

The hydrodynamic lubrication may be broadly classified under two groups, depending on the type of the lubricant, namely

(a) compressible hydrodynamic lubrication

(b) imcompressible hydrodynamic lubrication.

Each of the above two groups could be further subdivided into

(i) self acting bearing

(ii) externally pressurized bearing.

In a bearing different surface configurations are possible and prevalent, but the two most common configurations are

(i) the journal bearing

(ii) the thrust bearing.

In this thesis, the analysis will be confined to incompressible hydrodynamic lubrication and the assumed configuration will be a plane slider, representing a thrust bearing.

1.2 INERTIA EFFECTS AND TEMPERATURE VARIATION IN SLIDER BEARING

The Reynolds equation [1] is based on the assumption that the inertia forces are negligible compared to the viscous forces. The importance of the inertia terms relative to the viscous terms in the Navier-Stokes equations can be characterized by a dimensionless parameter referred to as the modified Reynolds number, Re^*.

In the operation of most of the bearings the Re^* is very small, therefore, the inertia effects could be easily ignored. However, for bearings operating at high speeds, particularly if the lubricant is of low kinematic viscosity, the Re^* may assume values near or exceeding unity. Therefore, the inertia forces become comparable with the viscous forces and the inertia effects may no longer be negligible.

BRAND [2] has shown that if the Reynolds number $\rho R\omega h_m/\mu$ is of

the same order as R/h_m, then the inertia forces will be of the same order as the viscous forces. However, if the Reynolds number becomes too large, turbulence will develop in the flow and the governing equations and the analysis will have to be modified suitably.

Another assumption made in the classical hydrodynamic lubrication is that the effects of temperature variation in the lubricant film are negligible. Therefore, the solution to the momentum equation is obtained for constant viscosity. However, in reality, energy is generated by viscous dissipation which results in the heating of the lubricant in the bearing, therefore decreasing the magnitude of the lubricant viscosity. Hence the solution obtained from the momentum equation for constant viscosity will be in error. In order to obtain an approximate solution in accordance with the actual operation of the bearing some mean viscosity value was employed in the solution of the momentum equation. The prediction of the mean viscosity value was made from the inlet and outlet temperature conditions.

Several authors have attempted to include the viscosity variation into account by allowing the viscosity variation in x-direction only. This results in a much simpler form of the momentum and the energy equations. These differential equations were solved simultaneously for the imposed boundary conditions.

The importance of the lateral temperature variation depends strongly on the thermal boundary conditions imposed on the film boundaries. If both boundaries are taken as adiabatic, all the energy generated by

viscous dissipation would have to be carried away by the fluid. If both boundaries were maintained at constant temperature, greater lateral temperature gradient would develop and heat transfer across both boundaries would occur.

1.3 REVIEW OF RELEVANT LITERATURE

The retention of inertia terms in the simplified Navier-Stokes equations for the slider bearing yields a non-linear differential equation. The influence of these non-linear inertia terms have been investigated by a number of authors adopting approximate methods.

SLEZKIN and TRAG [3] have proposed that the inertia terms are averaged across the film thickness. Therefore the inertia terms become a function of longitudinal direction only, and the resulting equation may be readily integrated.

Several authors [4,5] have employed this technique to obtain the solution for various bearing configurations. The averaged inertia approximation results in a parabolic velocity profile which need not always be applicable.

KAHLERT [6] developed a method of solving the differential equation in which it is assumed initially that the inertia forces are small in comparison with the viscous forces. Hence, their effects may be calculated as a small correction to the expression obtained from purely viscous consideration. The noticed difference between the results of Kahlert and the other authors can be traced to the approximation involved in assuming the inertia forces as a small fraction of the viscous

forces.

MILNE [7] has considered a solution to the complete equation of motion in cylindrical co-ordinates without using the thin film approximation. His technique consists of a series expansion of the stream function with reciprocal of kinematic viscosity as the expansion parameter. The zero order term in the series corresponds to the inertia-less case since $1/\nu \to 0$ for negligible inertia. In a later review he points out that this procedure is identical to that developed by Kahlert.

SNYDER [8] has obtained a more exact solution by taking into consideration the variation of the inertia effects across the film as well as in the direction of the flow. In this method the stream function approach is introduced. The stream function is expressed as a power series in δ which is the function of $h(x)$, whereas the coefficients are assumed to be a function of η which in turn depends on y co-ordinate. The coefficients are obtained by solving a set of differential equations. In presenting the solution SNYDER solved for only the first two terms of the power series. The treatment presented here using the numerical method indicates that it is necessary to include at least the first four terms to obtain more accurate results.

WOODHEAD and KETTLEBOROUGH [9] have obtained the solution of the differential equation by matrix methods. They have introduced the concept of field matrix which reduces the problem of determining the three field matrices [U], [V] and [P]. The solution is obtained by iterative procedure until the boundary conditions are satisfied. Generally,

the solution by matrix methods is long and cumbersome and takes substantial computer time.

HUNTER and ZIENKIEWICZ [10] considered the momentum and the energy equation in the coupled form for the isothermal, as well as the adiabatic bearing surfaces. The viscosity of the lubricant is regarded as a function of temperature. In their analysis they have omitted the inertia terms in the momentum equation and the $v\ \partial T/\partial y$ term in the energy equation. The solution to the differential equations is obtained by a finite difference method using an iterative procedure. The omission of inertia terms indicates that the solution is valid for very low modified Reynolds numbers.

The latest investigation was carried out by HAHN and KETTLEBOROUGH [11]. The coupled form of the momentum and the energy equation is considered for the isothermal surfaces. The viscosity of the lubricant is regarded as a function of temperature and pressure. They have retained the inertia terms in the momentum equation and the convective terms in the energy equation. The solution is obtained through matrix methods similar to the procedure mentioned in [9].

On examination of the literature pertinent to the incompressible hydrodynamic lubrication it seems apparent that the complete solution to the momentum and the energy equations is difficult due to their nonlinearity and coupled form. To achieve this object it seems necessary to introduce some form of transformation that would simplify the differential equations so that a simpler numerical solution is possible. It appears

that stream function methods have great potential in the study of lubrication films. It is felt that approach on these lines would further enhance the knowledge of film lubrication.

1.4 STATEMENT OF THE PROBLEM

The purpose of the present investigation is:

(a) to investigate the contribution of inertia forces for different modified Reynolds numbers,

(b) to consider the effect of convective terms in the energy equation, and

(c) to consider the temperature variation in the lubricating film of high and low viscosity oils.

The bearing configuration considered is a plane slider.

In obtaining the solution, the assumption of constant viscosity will be made. This results in uncoupling of the momentum equation from the energy equation so that an independent solution for the velocity distribution can be obtained.

The uncoupled momentum equation is then transformed through a stream function, yielding a set of ordinary differential equations. These equations are then solved numerically. The dependent variables such as pressure distribution, load capacity, and the shear stress at the moving surface are computed for various values of the modified Reynolds number, ($0.1 \leq Re^* \leq 1.0$). The increase in load capacity due to inertia effects is then computed for the above Re^*.

The solution to the energy equation is also obtained numerically

with and without convective terms. The temperature variation in the film and the temperature gradient at the moving and the stationary surfaces are calculated. In addition, these results have also been obtained for bearings using high and low viscosity oils. For these cases the energy equation considered included the convective terms.

The relevant results have been discussed in the final chapter.

CHAPTER II

THE GOVERNING EQUATIONS

2.1 GENERAL FORM OF THE EQUATIONS

Consider the Navier-Stokes equations and the energy equation [12] for the slider bearing geometry of Fig. 1.1.

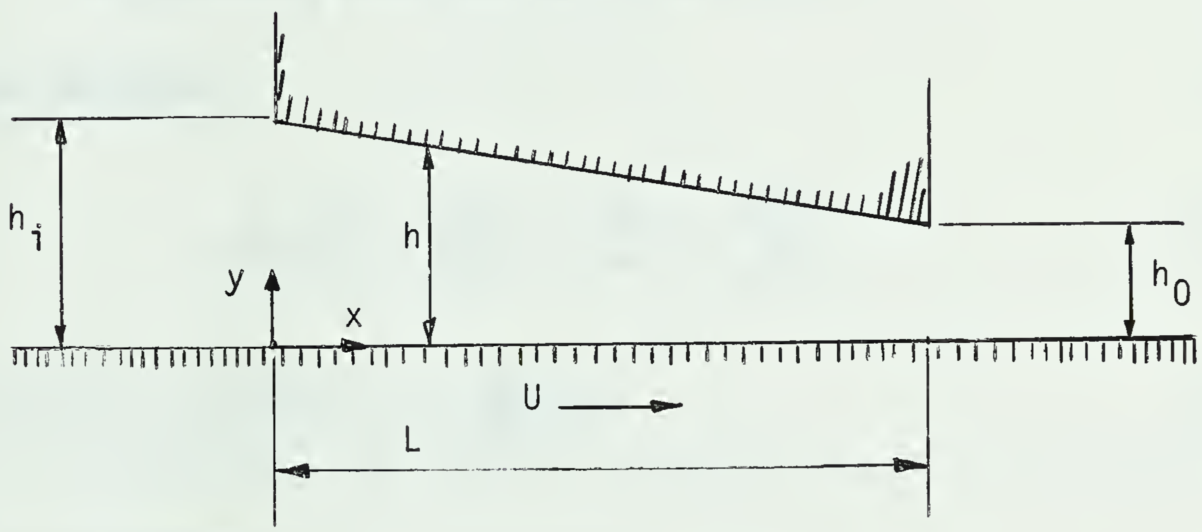

FIG. 1.1 SLIDER BEARING GEOMETRY

The Navier-Stokes equations and the energy equation are simplified under the following assumptions:

1. The problem is two dimensional.
2. Steady state has been reached.
3. The effects of thermal and elastic distortions are neglected.

4. The lubricant density and viscosity remains constant.

5. The thermal conductivity and specific heat of the lubricant are constant.

6. The flow is laminar.

7. Cavitation effects may be ignored.

8. Lubricating film is very thin, $h/L \ll 1$.

9. Order of magnitude considerations may be used to simplify the Navier-Stokes and the energy equations.

10. The inlet temperature of the lubricant may be taken constant.

The governing equations may be written as:

Momentum Equations

$$\rho\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = - \frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \tag{2.1}$$

$$\frac{\partial p}{\partial y} = 0 \; . \tag{2.2}$$

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \; . \tag{2.3}$$

Energy Equation

$$\rho \, C_p\left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \kappa \frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \; . \tag{2.4}$$

The boundary conditions

$$\left.\begin{aligned} u = U, \quad v = 0 \quad \text{at} \quad y = 0 \\ u = 0, \quad v = 0 \quad \text{at} \quad y = h(x) \end{aligned}\right\} \qquad (2.5)$$

$$\left.\begin{aligned} p = 0 \quad \text{at} \quad x = 0 \\ p = 0 \quad \text{at} \quad x = L \end{aligned}\right\} \qquad (2.6)$$

$$\left.\begin{aligned} T(0,y) = T_s \\ T(x,0) = T_s \\ T(x,h) = T_s \end{aligned}\right\} \qquad (2.7)$$

2.2 EQUATIONS IN DIMENSIONLESS FORM

The governing equations may also be written in the dimensionless form by introducing the following:

$$x = \bar{x}L$$

$$y = \bar{y}h$$

$$u = \bar{u}U$$

$$v = \bar{v}V$$

$$p = \bar{p}\, \rho U^2$$

$$T = \bar{T}\, T_s$$

The equations (2.1) through (2.7) can be written in the following dimensionless form:

$$\bar{u}\,\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\,\frac{\partial \bar{u}}{\partial \bar{y}} = -\,\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{Re^*}\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) \tag{2.8}$$

$$\frac{\partial \bar{p}}{\partial \bar{y}} = 0 \tag{2.9}$$

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2.10}$$

$$P\, Re^*\left(\bar{u}\,\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\,\frac{\partial \bar{T}}{\partial \bar{y}}\right) = EP\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \tag{2.11}$$

where the dimensionless parameters are defined

$$Re^* = \frac{UL}{\nu}\left(\frac{h_0}{L}\right)^2 \tag{2.12}$$

$$P = \frac{C_p\, \mu_s}{\kappa} \tag{2.13}$$

$$E = \frac{U^2}{C_p\, T_s} \tag{2.14}$$

The boundary conditions

$$\left.\begin{array}{l}\bar{u} = 1, \quad \bar{v} = 0 \quad \text{at} \quad \bar{y} = 0 \\ \\ \bar{u} = 0, \quad \bar{v} = 0 \quad \text{at} \quad \bar{y} = 1\end{array}\right\} \tag{2.15}$$

$$\left.\begin{array}{l}\bar{p} = 0 \quad \text{at} \quad \bar{x} = 0 \\ \\ \bar{p} = 0 \quad \text{at} \quad \bar{x} = 1\end{array}\right\} \tag{2.16}$$

$$\left.\begin{array}{l}\bar{T}(0,\bar{y}) = 1 \\ \\ \bar{T}(\bar{x},0) = 1 \\ \\ \bar{T}(\bar{x},1) = 1\end{array}\right\} \tag{2.17}$$

2.3 TRANSFORMATION OF THE MOMENTUM EQUATION

Let the stream function be defined by

$$u = \frac{\partial\psi}{\partial y} \tag{2.18}$$

$$v = -\frac{\partial\psi}{\partial x} \tag{2.19}$$

where after SNYDER [8]

$$\psi = K \sum_{n=o}^{\infty} f_n(\eta)\, \delta^n \qquad (2.20)$$

In the above $\delta = h/h_i$, $\eta = y/h$, and K is a non dimensionalizing constant to be specified later.

Expressing the appropriate velocity expressions in terms of the stream function gives

$$u = \frac{K}{h_i} \sum_{n=0}^{\infty} f_n' \, \delta^{n-1} \qquad (2.21)$$

$$\frac{\partial u}{\partial y} = \frac{K}{h_i^2} \sum_{n=0}^{\infty} f_n'' \, \delta^{n-2} \qquad (2.22)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{K}{h_i^3} \sum_{n=0}^{\infty} f_n''' \, \delta^{n-3} \qquad (2.23)$$

$$\frac{\partial u}{\partial x} = - \frac{Kh'}{h_i^2} \sum_{n=0}^{\infty} f_n' \, \delta^{n-2}$$

$$- \frac{K\eta h'}{h_i^2} \sum_{n=0}^{\infty} f_n'' \, \delta^{n-2}$$

$$+ \frac{Kh'}{h_i^2} \sum_{n=1}^{\infty} n\, f_n' \, \delta^{n-2} \qquad (2.24)$$

$$v = \frac{K\eta h'}{h_i} \sum_{n=0}^{\infty} f_n' \, \delta^{n-1}$$

$$- \frac{Kh'}{h_i} \sum_{n=1}^{\infty} n\, f_n \, \delta^{n-1} \qquad (2.25)$$

where $h' = \frac{dh}{dx}$ = constant.

On substituting (2.21) through (2.25) into the differential equation (2.1) the following is obtained:

$$- \frac{K^2 h' \rho}{\mu K} \left[\left(\sum_{n=0}^{\infty} f_n' \delta^{n-1} \right) \left(\sum_{n=0}^{\infty} f_n' \delta^{n-2} \right) \right]$$

$$+ \frac{K^2 h' \rho}{\mu K} \left[\left(\sum_{n=0}^{\infty} f_n' \delta^{n-1} \right) \left(\sum_{n=1}^{\infty} n f_n' \delta^{n-2} \right) \right]$$

$$- \frac{K^2 h' \rho}{\mu K} \left[\left(\sum_{n=0}^{\infty} f_n'' \delta^{n-2} \right) \left(\sum_{n=1}^{\infty} n f_n \delta^{n-1} \right) \right]$$

$$+ \frac{h_i^3}{\mu K} \frac{\partial p}{\partial x} = \sum_{n=0}^{\infty} f_n''' \delta^{n-3} \tag{2.26}$$

Let the constant K be such that

$$\frac{K^2 h' \rho}{\mu K} = 1$$

which yields

$$K = \frac{\nu}{h'}$$

and consequently (2.26) becomes

$$- \left(\sum_{n=0}^{\infty} f_n' \, \delta^{n-1} \right) \left(\sum_{n=0}^{\infty} f_n' \, \delta^{n-2} \right)$$

$$+ \left(\sum_{n=0}^{\infty} f_n' \, \delta^{n-1} \right) \left(\sum_{n=1}^{\infty} n \, f_n' \, \delta^{n-2} \right)$$

$$- \left(\sum_{n=0}^{\infty} f_n'' \, \delta^{n-2} \right) \left(\sum_{n=1}^{\infty} n \, f_n \, \delta^{n-1} \right)$$

$$+ \frac{h_i^3}{\mu K} \frac{\partial p}{\partial x} = \sum_{n=0}^{\infty} f_n''' \delta^{n-3} \tag{2.27}$$

The pressure gradient term in equation (2.27) is also assumed to have the following series from expansion [8]

$$\frac{h_i^3}{\mu} \frac{\partial p}{\partial x} = \frac{h_i^3}{h^3} (C_0 + C_1\delta + C_2\delta^2 + C_3\delta^3 + \ldots) \tag{2.28}$$

where C's are constants. Combining equation (2.28) and (2.27) a set of differential equations is found of which the following are the first four

$$\delta^{-3}: \qquad f_0''' + f_0' f_0' = \frac{1}{K} C_0 \tag{2.29}$$

$$\delta^{-2}: \qquad f_1''' + f_0'' f_1 + f_0' f_1' = \frac{1}{K} C_1 \tag{2.30}$$

$$\delta^{-1}: \qquad f_2''' + 2f_0'' f_2 + f_1'' f_1 = \frac{1}{K} C_2 \tag{2.31}$$

$$\delta^{0}: \qquad f_3''' - f_0' f_3' + 3f_0'' f_3 - f_1' f_2' + 2f_1'' f_2 + f_1 f_2''$$

$$= \frac{1}{K} C_3 \tag{2.32}$$

2.4 ASSOCIATED BOUNDARY CONDITIONS

The boundary conditions to be satisfied by the momentum equation are (2.5) to (2.6).

From the boundary condition (2.5), a set of conditions for function f are obtained by introducing this boundary condition into the equations (2.21) and (2.25). These are the conditions that are to be satisfied by the differential equations (2.29) to (2.32).

$$\left.\begin{aligned} f_0(0) &= 0\ , \quad f_0'(0) = 0 \\ f_0(1) &= \frac{m}{\rho K}\ , \quad f_0'(1) = 0 \end{aligned}\right\} \tag{2.33}$$

$$\left.\begin{aligned} f_1(0) &= 0\ , \quad f_1'(0) = \frac{h_i U}{K} \\ f_1(1) &= 0\ , \quad f_1'(1) = 0 \end{aligned}\right\} \tag{2.34}$$

and

$$\left.\begin{aligned} f_n(0) &= 0\ , \quad f_n'(0) = 0 \\ f_n(1) &= 0\ , \quad f_n'(1) = 0 \end{aligned}\right\} \tag{2.35}$$

where

$$n = 2, 3, \ldots$$

Each of the equations (2.29) through (2.32) for the f-functions is of third order, and thus three boundary conditions will be required for

obtaining the solution. However, each equation also involves an unknown constant C which will require one more condition on function f. This necessary condition for C_0, C_1 and C_n will be $f_0(1)$, $f_1(1)$ and $f_n(1)$ respectively. The C's, which depend on m as seen from equation (2.33) will determine the pressure gradient. The pressure then will be obtained by integration of expression (2.28).

CHAPTER III

SOLUTION TO THE GOVERNING EQUATIONS

3.1 NUMERICAL SOLUTION TO THE MOMENTUM EQUATION

The transformation of the momentum equation (2.1) which includes the inertia terms for the slider bearing, yields a set of differential equations (2.29) to (2.32) along with the boundary conditions (2.33) to (2.35). The methods of solutions to these differential equations with their respective boundary conditions are described.

(a) Solution to the first term in stream function expansion (f_0)

The differential equation (2.29) for f_0 is of third order and non linear. Its non linearity made it difficult to obtain a closed form solution. It was therefore, decided to apply some numerical technique to obtain the solution.

The differential equation (2.29) and the boundary conditions (2.33) are reducible to an equivalent initial value problem involving the solution of a system of first order ordinary differential equations, subjected to prescribed initial conditions. The initial conditions to be imposed on f_0 and f_0' are obvious, however, the initial value of f_0'' is not readily prescribed. Rather a trial and error method is adopted.

To begin the computation some value of C_0 is assumed. Then the differential equation is solved with the various assumed values of $f''(0)$

until the boundary conditions $f_0(0)$, $f_0'(0)$, and $f_0'(1)$ are satisfied. It is found that the relation between $f_0''(0)$ and $f_0'(1)$ is approximately linear which results in a very good approximation of $f''(0)$ after only two iterations. The final value of $f''(0)$ is found by iteratively correcting its previously obtained value.

After solving the differential equation for the assumed C_0 the value of $f_0(1)$ is compared with the respective boundary conditions. In case of disagreement a new value of C_0 is assumed and the above procedure is repeated until the boundary conditions are satisfied. It was found that the relation between C_0's and the values of $f_0(1)$ obtained in the above procedure is also approximately linear. Therefore, a better approximation of C_0 was made possible. The final value of C_0 is found by iteratively correcting its previously obtained value.

Gill's variation of the Runge-Kutta fourth order method is employed as an integration procedure. The subroutine DRKGS from IBM System/360 scientific subroutine package is employed to obtain the solution.

(b) Solution to the second term in stream function expansion (f_1)

The differential equation (2.30) for f_1 is of third order and linear, and the coefficients involving f_0's has already been obtained.

The solution to this equation with the boundary conditions (2.34) is obtained with the following numerical procedure.

The derivatives in the differential equation are approximated

by the finite difference approximations. The resulting equation can be represented by the recurrance relation as suggested by Rodkiewicz and Reshotko [13]

$$S_n = R_n S_{n+1} + D_n \tag{3.1}$$

where

$$R_n = \frac{-a_n}{(b_n + c_n R_{n-1} + d_n R_{n-1} R_{n-2})} \tag{3.2}$$

and

$$D_n = \frac{e_n - (c_n D_{n-1} + d_n R_{n-2} D_{n-1} + d_n D_{n-2})}{(b_n + c_n R_{n-1} + d_n R_{n-1} R_{n-2})} \tag{3.3}$$

where a_n, b_n, c_n, d_n and e_n are defined in Appendix A.

For n = 1 we have from the boundary condition (2.34)

$$R_1 = S_1 = D_1 = 0 \tag{3.4a}$$

and

$$\frac{\partial s}{\partial \eta} = \frac{1}{2H} (-S_1+4S_2-S_3) = \frac{h_i U}{K} \tag{3.4b}$$

On considering equations (3.4b) and (3.1) for n = 2 we obtain

$$R_2 = 0.25 \;, \quad D_2 = \frac{h_i U}{K} \times \frac{H}{2} \tag{3.4c}$$

For $n \geq 3$ one can find from (3.2) and (3.3) all R_n and D_n respectively as we will have R_{n-2}, R_{n-1}, D_{n-2} and D_{n-1}. With these values one can obtain from (3.1) variable S_n provided we have S_{n+1}. Let n = N represent the location for which the upper boundary condition prevails. We know that at this location $S'(1) = 0$ and therefore

$$\frac{\partial S}{\partial \eta} = \frac{1}{2H} (-3S_1+4S_2-S_3) = 0 \tag{3.5}$$

and consequently

$$S_{N-2} = 4S_{N-1} - 3S_N \tag{3.6}$$

Also from (3.1) we obtain

$$S_{N-1} = R_{N-1}S_N + D_{N-1} \tag{3.7}$$

and

$$S_{N-2} = R_{N-2}S_{N-1} + D_{N-2} \tag{3.8}$$

Substituting (3.6) into (3.8) we have

$$S_{N-1} = \frac{3S_N + D_{N-2}}{4 - R_{N-2}} \tag{3.9}$$

and using (3.9) in (3.7) we finally obtain

$$S_N = \frac{D_{N-1} - \dfrac{D_{N-2}}{4 - R_{N-2}}}{\dfrac{3}{4 - R_{N-2}} - R_{N-1}} \tag{3.10}$$

which constitutes the starting point in the use of expression (3.1).

To begin the computation, some value of the unknown constant C_1 in the differential equation (2.30) is assumed. The assumed magnitude is then varied until all the boundary conditions are satisfied. These

computations were done on IBM System/360.

(c) Solution to the third and the fourth terms in stream function expansion (f_2 and f_3)

The differential equations (2.31) and (2.32) are of third order and linear. The solutions to these equations with their respective boundary conditions are obtained by using the numerical procedure as described in section 3.1b.

(d) Pressure distribution

The solution of differential equations (2.29) to (2.32) yields the values of C_0, C_1, C_2, and C_3 which are now inserted in the equation (2.28). Hence giving the expression for the pressure gradient along the length of the bearing. This expression is integrated using Simpson's Integration Rule, resulting in a pressure distribution. Now the obtained value of pressure at the outlet is compared with the boundary condition (2.6) and if these two values agree within the specified limit, then we have obtained the solution to the problem. In case of disagreement, a new assumption of the mass flow is made and the above procedure is repeated.

3.2 NUMERICAL SOLUTION TO THE ENERGY EQUATION

The solution to the energy equation (2.11) with the boundary conditions (2.17) is obtained by the method of finite differences. The derivatives are replaced by the central finite difference approximations, and the terms are arranged to obtain (see Appendix B) the equation in the following form.

$$a_{m,n}\,\overline{T}_{m,n-1} + b_{m,n}\,\overline{T}_{m,n} + C_{m,n}\,\overline{T}_{m,n+1} = d_{m,n} \tag{3.11}$$

where $a_{m,n}$, $b_{m,n}$, $c_{m,n}$ and $d_{m,n}$ are defined in Appendix B.

The boundary conditions become

$$\overline{T}_{m,1} = 1 \tag{3.12}$$

$$\overline{T}_{m,N} = 1 \tag{3.13}$$

$$\overline{T}_{1,n} = 1 \tag{3.14}$$

In order to obtain the solution the grid presented below was employed. The spacing interval in $\overline{x}$ and $\overline{y}$ direction was taken 0.1 and 0.05 respectively.

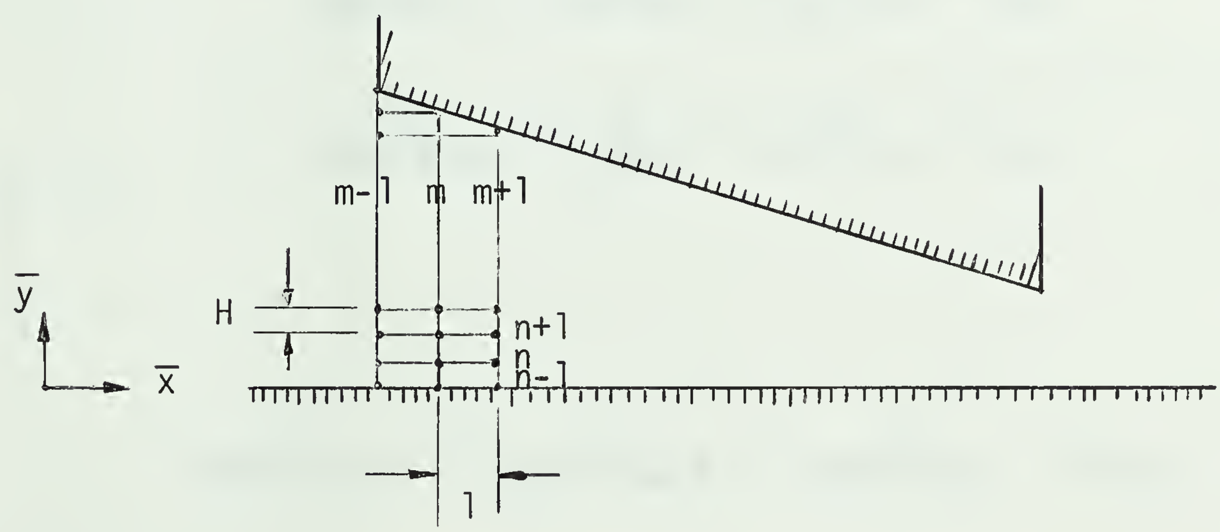

FIG. 3.1 GRID NETWORK

Equation (3.11) at location m = 2 yields

$$a_{2,n}\,\overline{T}_{2,n-1} + b_{2,n}\,\overline{T}_{2,n} + c_{2,n}\,\overline{T}_{2,n+1} = d_{2,n} \tag{3.15}$$

and the boundary conditions become

$$\overline{T}_{2,1} = 1 \tag{3.16}$$

$$\overline{T}_{2,N} = 1 \tag{3.17}$$

This yields a linear system of N-2 equations with N-2 unknowns namely $\overline{T}_{2,2}$, $\overline{T}_{2,3}$, ... $\overline{T}_{2,N-1}$. The associated coefficients are known from the solution of the momentum equation and the initial condition (3.14).

$$\begin{array}{c}
a_{2,2}\overline{T}_{2,1} + b_{2,2}\overline{T}_{2,2} + c_{2,2}\overline{T}_{2,3} = d_{2,2} \\
a_{2,3}\overline{T}_{2,2} + b_{2,3}\overline{T}_{2,3} + c_{2,3}\overline{T}_{2,4} = d_{2,3} \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
a_{2,N-2}\overline{T}_{2,N-3} + b_{2,N-2}\overline{T}_{2,N-2} + c_{2,N-2}\overline{T}_{2,N-1} = d_{2,N-2} \\
a_{2,N-1}\overline{T}_{2,N-2} + b_{2,N-1}\overline{T}_{2,N-1} + c_{2,N-1}\overline{T}_{2,N} = d_{2,N-1}
\end{array} \tag{3.18}$$

Restating the equation (3.18) in the matrix form, the following is obtained:

$$\begin{bmatrix} b_{2,2} & c_{2,2} & & & \\ a_{2,3} & b_{2,3} & c_{2,3} & & \\ & & \cdot & & \\ & & & \circ & \\ & & a_{2,N-2} & b_{2,N-2} & c_{2,N-2} \\ & & & a_{2,N-1} & b_{2,N-1} \end{bmatrix} \begin{bmatrix} \overline{T}_{2,2} \\ \overline{T}_{2,3} \\ \overline{T}_{2,4} \\ \cdot \\ \overline{T}_{2,N-2} \\ \overline{T}_{2,N-1} \end{bmatrix} = \begin{bmatrix} d_{2,2}-a_{2,2} \\ d_{2,3} \\ d_{2,4} \\ \circ \\ d_{2,N-2} \\ d_{2,N-1}-c_{2,N-1} \end{bmatrix} \quad (3.19)$$

The above matrix is a tridiagonal and of order N-2. Gauss's elimination method is employed to obtain the solution.

Once $\overline{T}_{2,n}$ is obtained equation (3.11) can be solved for $m = 3,4,5,\ldots$ using the same procedure.

The subroutine DGELB from IBM System/360 scientific subroutine packages is used for computations.

CHAPTER IV

DISCUSSION AND CONCLUSIONS

4.1 INERTIA EFFECTS

The momentum equation (2.1) is solved for Re^* = 0.1, 0.25, 0.5, 0.75 and 1.0. For comparison the computations were made for the case of neglected inertia terms. The dependent variables, pressure, load capacity and shear stress at the moving component are obtained. The tabulated results and plotted graphs are given in Appendix C.

Tables 4.1 and 4.2 give the values of $\overline{p}$ and $\overline{W}$ at different Re^* with and without inertia terms respectively. Table 4.3 shows the increase in load capacity due to the presence of inertia terms.

Figures 4.1 and 4.2 shows the pressure distribution with and without the inertia terms for Re^* = 0.25 and 1.0 respectively. Figure 4.3 gives the pressure computed with the inertia terms for Re^* = 0.25, 0.5, 0.75 and 1.0. It appears from these curves that the location of the maximum pressure is approximately independent of Re^* and the pressure increases as the Re^* increases.

Figure 4.4 shows the shear stress distribution at the moving component for Re^* = 0.25, 0.5, 1.0. The shear stress increases with the increase of Re^*.

In Fig. 4.5 the velocity distribution has also been plotted at different sections along the bearing for Re^* = 0.25, 0.5 and 1.0. It

is noted that the curves of the dimensionless velocity become more concave with the increase of Re^*.

The load capacity and the increase in load capacity due to presence of inertia terms are plotted in Fig. 4.6. The maximum increase in the load capacity was found to be 8.9% at $Re^* = 1.0$.

In order to make a comparison with the work of Kahlert [6], and Woodhead and Kettleborough [9], the dimensionless load capacity is redefined according to [9]. The pertinent curves are plotted in Fig. 4.7. It can be seen that the present results are in very close agreement with those of Woodhead and Kettleborough.

A similar computation was also carried out by considering only the first two terms of expression (2.28). Table 4.4 gives the pressure distribution and the load capacity for various Re^*. Fig. 4.8 shows the curves of pressure distributions calculated considering the first two terms only as well as considering the first four terms of the expression (2.28) for $Re^* = 1.0$. Figure 4.9 shows the increase in load capacity percentage-wise due to the additional contribution of the 3rd and the 4th term of the series. It is noted that the contribution of the last two terms of the expression (2.28) is approximately 74% at $Re^* = 1.0$, hence their effects cannot be ignored as has been suggested by SNYDER [8]. Thus it would seem that it may be necessary to include at least four terms in the pressure expression (2.28) to obtain more accurate results.

4.2 CONVECTION EFFECTS

The dimensionless form of energy equation (2.11) shows that the solution of the differential equation depends upon three important parameters - the modified Reynolds number, the Prandtl number and the Eckert number. For computations the lubricant with P = 122 is selected here. The differential equations are solved with and without convective terms for $Re^* = 0.25$, and 1.0. The corresponding values of E for $T_s = 150°F$ are 0.000473 and 0.00757 respectively.

Figures 4.10 and 4.11 show the temperature distribution in the fluid film without the convective terms at different locations along the bearing. The maximum temperature rise was found to be 1.3°F and 20.0°F for $Re^* = 0.25$ and 1.0 respectively.

The temperature distribution in the fluid film with the convective terms is plotted in Figs. 4.12 and 4.13. The maximum temperature rise was found to be 0.4 and 2.0°F for $Re^* = 0.25$ and 1.0 respectively.

Figures 4.14 and 4.15 show the temperature gradient with and without the convective terms at the moving and stationary components for $Re^* = 0.25$ and 1.0 respectively.

The results i.e., the temperature distribution and the temperature gradient obtained with and without the convective terms for $Re^* = 1.0$ indicates that

(i) the maximum temperature rise in the film is approximately 10 times higher for the case of neglected convective terms,

(ii) the minimum error, due to neglecting convective terms, in the

temperature gradient at the moving and stationary surfaces is approximately 1255% and 1411% respectively. This minimum error is obtained by the following expression

$$\frac{\left(\frac{\partial \bar{T}}{\partial y}\right)_{\text{Without Convective Terms}} - \left(\frac{\partial \bar{T}}{\partial y}\right)_{\text{With Convective Terms}}}{\left(\frac{\partial \bar{T}}{\partial y}\right)_{\text{With Convective Terms}}}$$

Thus it would seem that the effect of convective terms in the analysis of hydrodynamic bearing is by no means negligible and their omission, particularly at high Re^* may introduce appreciable errors.

4.3 TEMPERATURE VARIATION AT HIGH PRANDTL NUMBERS

The momentum equation (2.8) and the energy equation (2.11) are also solved for high viscosity lubricating oils. Figure 4.16 shows the temperature variation for P = 2174.6. The Re^* and E are 0.04872 and 0.0082 respectively. The maximum temperature rise in the film is found to be 34°F.

Furthermore a similar computation were also carried out for a light lubricating oil, P = 347.8. The Re^* and E being 0.305 and .0082 respectively. The temperature variation in the film is plotted in Fig. 4.17. The maximum temperature rise was found to be 6.0°F.

In comparing the temperature distribution obtained for the above two cases it is noted that the lubricant of high Prandtl number develops larger temperature variation in the film.

4.4 CONCLUDING REMARKS

In obtaining the solution of the differential equations there were certain assumptions made vide Chapter II. It is noted from the present results that when the viscosity of the lubricant is high, and particularly if Re^* is very small, large temperature variations develop within the film. For such cases it would be necessary to incorporate viscosity dependence on temperature in the differential equations. This would bring additional difficulties in solving the differential equations as the momentum equations will be now coupled with the energy equation through viscosity.

A further assumption made was that the boundaries are rigid. In practise this may not be true for the case of bearings developing very high pressures and temperatures. For such cases it would be necessary to include the equations of elasticity for the boundaries in addition to the differential equations governing the fluid flow.

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APPENDIX 'A'

RECURRENCE RELATIONS INVOLVING THE 4-POINT COMPUTATIONAL MOLECULE

The differential equation (2.30) may be written for simplicity in the following form

$$S_n''' + A_n S_n + B_n S_n' = G \tag{A.1}$$

where S is a dummy variable, A_n and B_n are known coefficients and G is a constant.

The following shape of the computational molecule will be used in the numerical method.

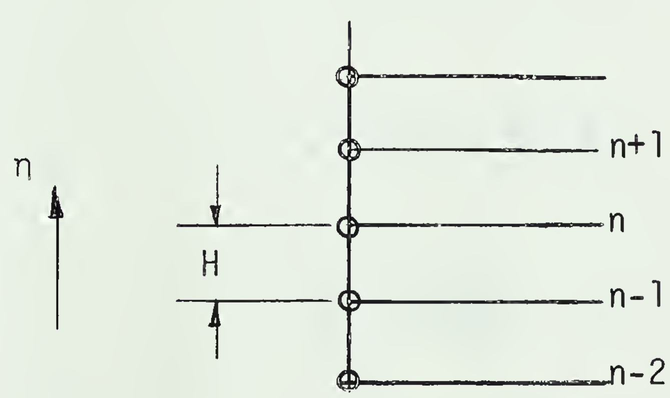

FIG. A.1

Let H be the step size in η direction. Then [13] we may write

$$S_n' = \frac{1}{2H}(S_{n+1} - S_{n-1}) \tag{A.2}$$

$$S_n''' = \frac{1}{H^3}(S_{n+1} - 3S_n + 3S_{n-1} - S_{n-2}) \tag{A.3}$$

Substituting (A.2) and (A.3) into the equation (A.1) and after re-arranging we obtain

$$\left(1 + \frac{H^2}{2} B_n\right) S_{n+1} + (H^3 A_n - 3) S_n + \left(3 - \frac{H^2}{2} B_n\right) S_{n-1}$$

$$- S_{n-2} = H^3 G \tag{A.4}$$

The expression (A.4) can also be written in the following general form

$$a_n S_{n+1} + b_n S_n + c_n S_{n-1} + d_n S_{n-2} = e_n \tag{A.5}$$

where

$$a_n = \left(1 + \frac{H^2}{2} B_n\right) \tag{A.6}$$

$$b_n = (H^3 A_n - 3) \tag{A.7}$$

$$c_n = \left(3 - \frac{H^2}{2} B_n\right) \tag{A.8}$$

$$d_n = -1.0 \tag{A.9}$$

$$e_n = H^3 G \tag{A.10}$$

Considering the boundary conditions in conjunction with (A.5) a recurrance formula of the following from can be developed [13]

$$S_n = R_n S_{n+1} + D_n$$

APPENDIX 'B'

ENERGY EQUATION IN FINITE DIFFERENCE FORM

The energy equation (2.11) is approximated using a 4-point as well as a 5-point computational molecule. The 4-point molecule approximation is only employed at m = 2 location, and for m = 3 onwards the 5-point molecule is used for better approximation.

B.1 4-POINT FINITE DIFFERENCE APPROXIMATION

For the 4-point finite difference approximation the following computational molecule was considered

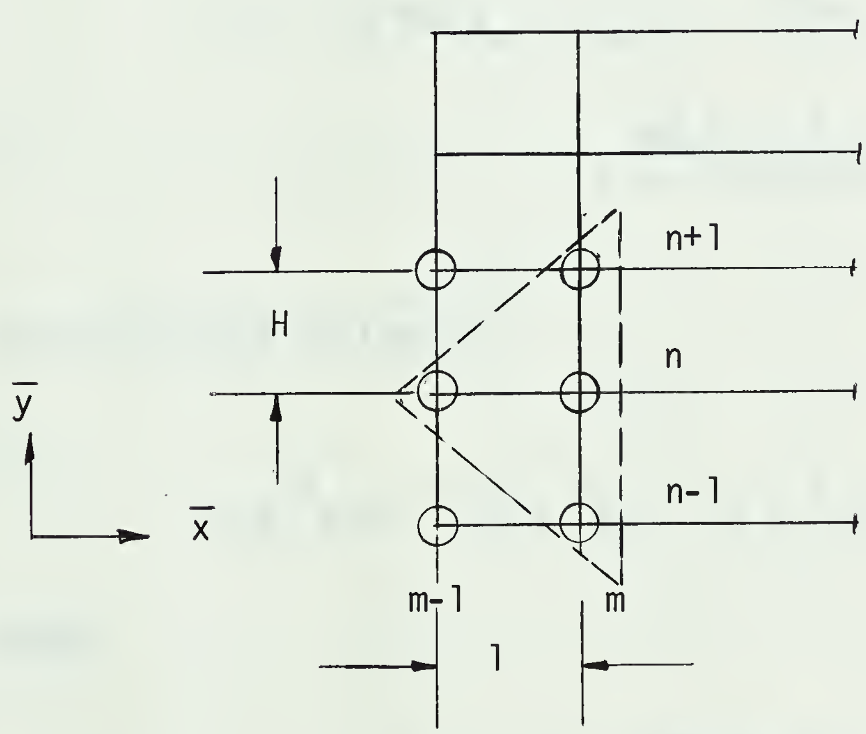

FIG. B.1

Let l and H be the elementary step magnitude in $\bar{x}$ and $\bar{y}$ direction respectively. Then [14] we may write

$$\frac{\partial \bar{T}}{\partial \bar{x}} = \frac{1}{l} (\bar{T}_{m-1,n} - \bar{T}_{m,n}) \tag{B.1}$$

$$\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{1}{2H} (\bar{T}_{m,n+1} - \bar{T}_{m,n-1}) \tag{B.2}$$

$$\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} = \frac{1}{H^2} (\bar{T}_{m,n+1} - 2\bar{T}_{m,n} + \bar{T}_{m,n-1}) \tag{B.3}$$

Substituting (B.1) through (B.3) in the energy equation (2.11) we obtain

$$- (1 + \frac{H}{2} PRe^* \bar{v}_{m,n}) \bar{T}_{m,n-1} + (2 + \frac{H^2 PRe^* \bar{u}_{m,n}}{l}) \bar{T}_{m,n}$$
$$+ (-1 + \frac{H}{2} PRe^* \bar{v}_{m,n}) \bar{T}_{m,n+1} = H^2[PE (\frac{\partial \bar{u}_{m,n}}{\partial \bar{y}})^2$$
$$+ \frac{PRe^* \bar{u}_{m,n} \bar{T}_{m,n}}{l}] \tag{B.4}$$

which could be written as

$$a_{m,n} \bar{T}_{m,n-1} + b_{m,n} \bar{T}_{m,n} + c_{m,n} \bar{T}_{m,n+1} = d_{m,n} \tag{B.5}$$

where

$$a_{m,n} = - (1 + \frac{H}{2} PRe^* \bar{v}_{m,n}) \tag{B.6}$$

$$b_{m,n} = (2 + \frac{H^2 \ PRe^*}{1} \bar{u}_{m,n}) \quad (B.7)$$

$$c_{m,n} = - (1 - \frac{H}{2} PRe^* \ \bar{v}_{m,n}) \quad (B.8)$$

$$d_{m,n} = H^2[PE \ (\frac{\partial \bar{u}_{m,n}}{\partial \bar{y}})^2 + \frac{PRe^* \ \bar{u}_{m,n} \ \bar{T}_{m,n}}{1}] \quad (B.9)$$

B.2 5-POINT FINITE DIFFERENCE APPROXIMATION

For the computational molecules indicated below in Fig. B.2 we may write

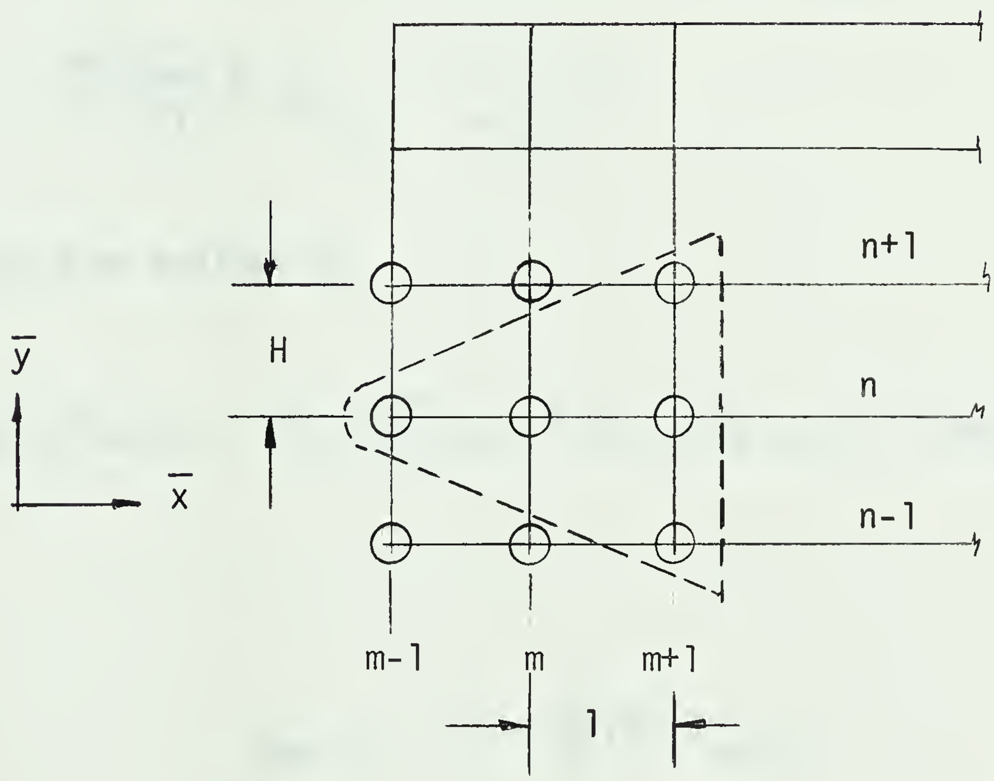

FIG. B.2

$$\frac{\partial \bar{T}}{\partial \bar{x}} = \frac{1}{2l}(3\bar{T}_{m+1,n} - 4\bar{T}_{m,n} + \bar{T}_{m-1,n}) \tag{B.10}$$

$$\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{1}{2H}(\bar{T}_{m+1,n+1} - \bar{T}_{m+1,n-1}) \tag{B.11}$$

$$\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} = \frac{1}{H^2}(\bar{T}_{m+1,n+1} - 2\bar{T}_{m,n} + \bar{T}_{m+1,n-1}) \tag{B.12}$$

Substituting (B.10) through (B.12) into the energy equation we obtain

$$-(1 + \frac{H}{2} PRe^* \bar{v}_{m+1,n}) \bar{T}_{m+1,n-1} + (2 + \frac{3H^2 PRe^* \bar{u}_{m+1,n}}{2l})\bar{T}_{m+1,n}$$

$$+ (-1 + \frac{H}{2} PRe^* \bar{v}_{m+1,n}) \bar{T}_{m+1,n+1} = H^2[PE(\frac{\partial \bar{u}_{m+1,n}}{\partial \bar{y}})^2$$

$$+ \frac{PRe^* \bar{u}_{m+1,n}}{2l} (4\bar{T}_{m,n} - \bar{T}_{m-1,n})] \tag{B.13}$$

which could be written as

$$a_{m+1,n}\bar{T}_{m+1,n-1} + b_{m+1,n}\bar{T}_{m+1,n} + c_{m+1,n}\bar{T}_{m+1,n+1} = d_{m+1,n} \tag{B.14}$$

where

$$a_{m+1,n} = -(1 + \frac{H}{2} PRe^* \bar{v}_{m+1,n}) \tag{B.15}$$

$$b_{m+1,n} = (2 + \frac{3H^2 PRe^* \bar{u}_{m+1,n}}{2l}) \tag{B.16}$$

$$c_{m+1,n} = -\left(1 - \frac{H}{2} PRe^{*} \bar{v}_{m+1,n}\right) \tag{B.17}$$

$$d_{m+1,n} = H^2\left[PE \left(\frac{\partial \bar{u}_{m+1,n}}{\partial \bar{y}}\right)^2 + PRe^{*} \frac{\bar{u}_{m+1,n}}{2l} \left(4\bar{T}_{m,n} - \bar{T}_{m-1,n}\right)\right] \tag{B.18}$$

APPENDIX C

TABLES AND CURVES FOR

CHAPTER IV

TABLE 4.1

PRESSURE DISTRIBUTION WITH INERTIA TERMS

Re^*	$\bar{p} = p/\rho U^2$											$\bar{W} = \int_0^1 \bar{p} d\bar{x}$
0.1	0.0	0.5185	1.0270	1.5110	1.9492	2.3098	2.5450	2.5822	2.3084	1.5447	0.0	1.6411
0.25	0.0	0.2097	0.4152	0.6108	0.7877	0.9331	1.0277	1.0421	0.9308	0.6215	0.0	0.6664
0.5	0.0	0.1071	0.2120	0.3118	0.4020	0.4762	0.5244	0.5318	0.4753	0.3181	0.0	0.3463
0.75	0.0	0.0728	0.1442	0.2119	0.2732	0.3234	0.3560	0.3608	0.3223	0.2155	0.0	0.2310
1.0	0.0	0.0557	0.1103	0.1621	0.2088	0.2471	0.2719	0.2755	0.2460	0.1644	0.0	0.1765
$\bar{x} \rightarrow$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	

Table prepared for $h_r = 2$

TABLE 4.2

PRESSURE DISTRIBUTION WITHOUT INERTIA TERMS

Re^*	$\bar{p} = p/\rho U^2$											$\bar{W} = \int_0^1 \bar{p}\,d\bar{x}$
0.1	0.0	0.5091	1.0085	1.4840	1.9147	2.2693	2.5008	2.5378	2.2693	1.5191	0.0	1.6225
0.25	0.0	0.2035	0.4031	0.5931	0.7653	0.9070	0.9995	1.0143	0.9070	0.6071	0.0	0.6485
0.5	0.0	0.1017	0.2015	0.2965	0.3826	0.4535	0.4997	0.5071	0.4535	0.3035	0.0	0.3242
0.75	0.0	0.0678	0.1343	0.1977	0.2551	0.3023	0.3331	0.3381	0.3023	0.2023	0.0	0.2161
1.0	0.0	0.0508	0.1007	0.1482	0.1913	0.2267	0.2498	0.2535	0.2267	0.1517	0.0	0.1621
$\bar{x} \rightarrow$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	

Table prepared for $h_r = 2$

TABLE 4.3

INCREASE IN LOAD CAPACITY

Re^*	% Increase in Load Capacity Due to Presence of Inertia Terms
0.1	1.764
0.25	2.764
0.5	4.676
0.75	6.89
1.0	8.9

Table prepared for $h_r = 2$

TABLE 4.4

PRESSURE DISTRIBUTION CONSIDERING THE FIRST

TWO TERMS OF EXPRESSION (2.28)

Re^*	$\bar{p} = p/\rho U^2$											$\bar{W} = \int_0^1 \bar{p}d\bar{x}$
0.1	0.0	0.5140	1.0175	1.497	1.928	2.280	2.5210	2.5407	2.2883	1.5310	0.0	1.6368
0.25	0.0	0.2061	0.4084	0.6009	0.7753	0.9188	1.0125	1.0274	0.9185	0.6145	0.0	0.6568
0.5	0.0	0.1033	0.2047	0.3012	0.3886	0.4605	0.5074	0.5147	0.4599	0.3072	0.0	0.3290
0.75	0.0	0.0691	0.1370	0.2015	0.2600	0.3082	0.3397	0.3447	0.3082	0.2064	0.0	0.22041
1.0	0.0	0.0520	0.1030	0.1516	0.1956	0.2319	0.2556	0.2594	0.2321	0.1555	0.0	0.1659
$\bar{x} \rightarrow$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	

Table prepared for $h_r = 2$

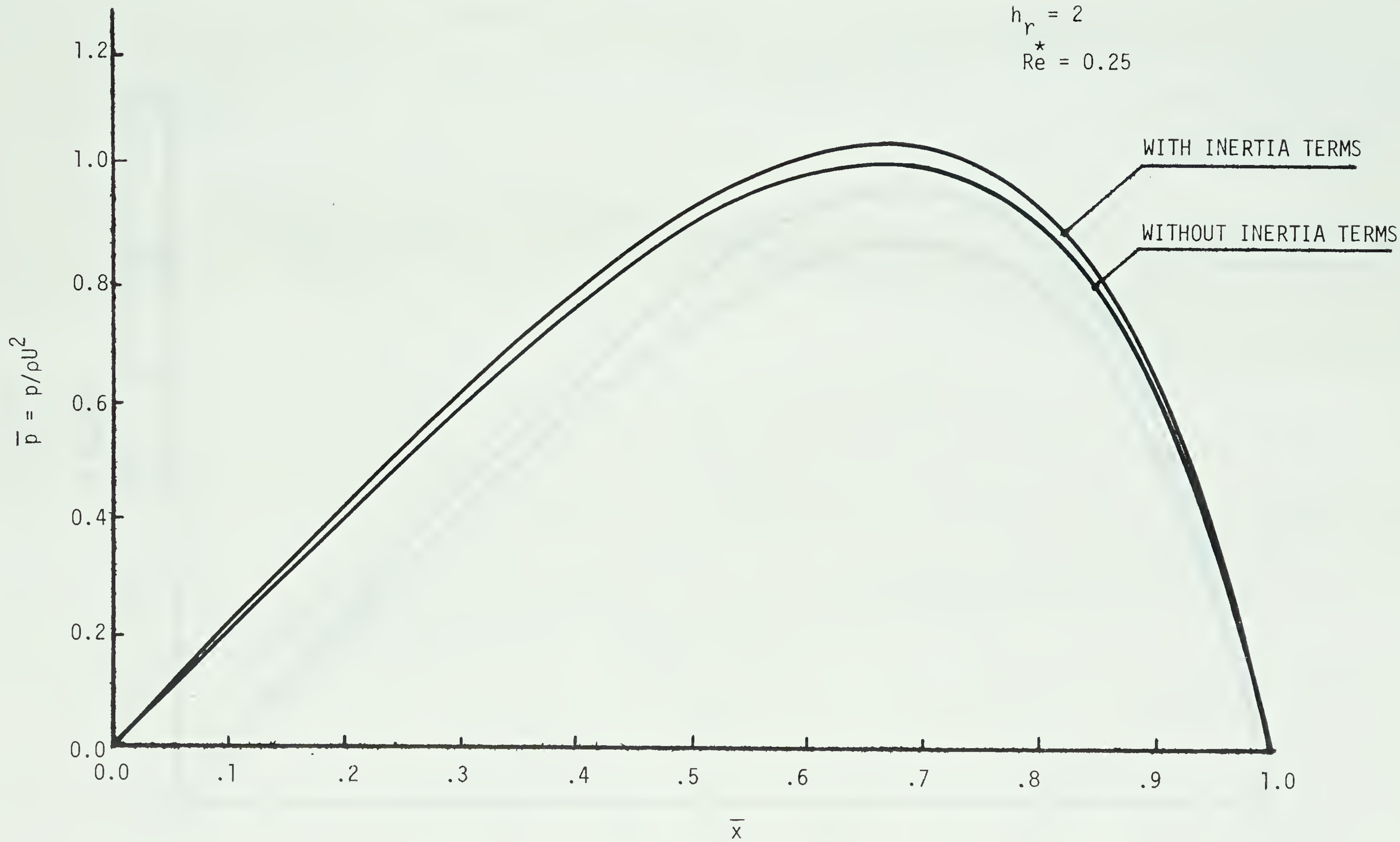

FIG. 4.1 PRESSURE DISTRIBUTION

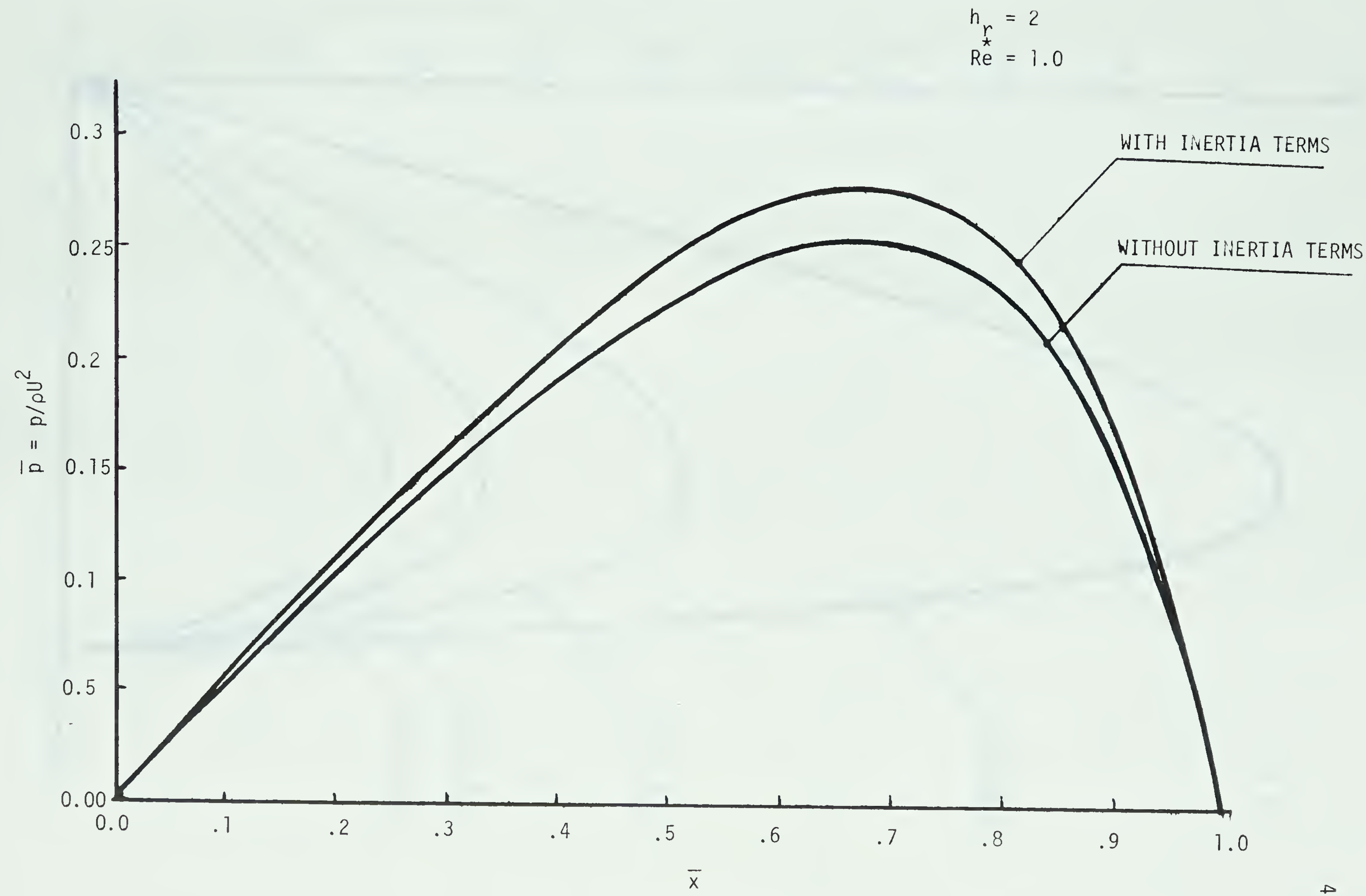

FIG. 4.2 PRESSURE DISTRIBUTION

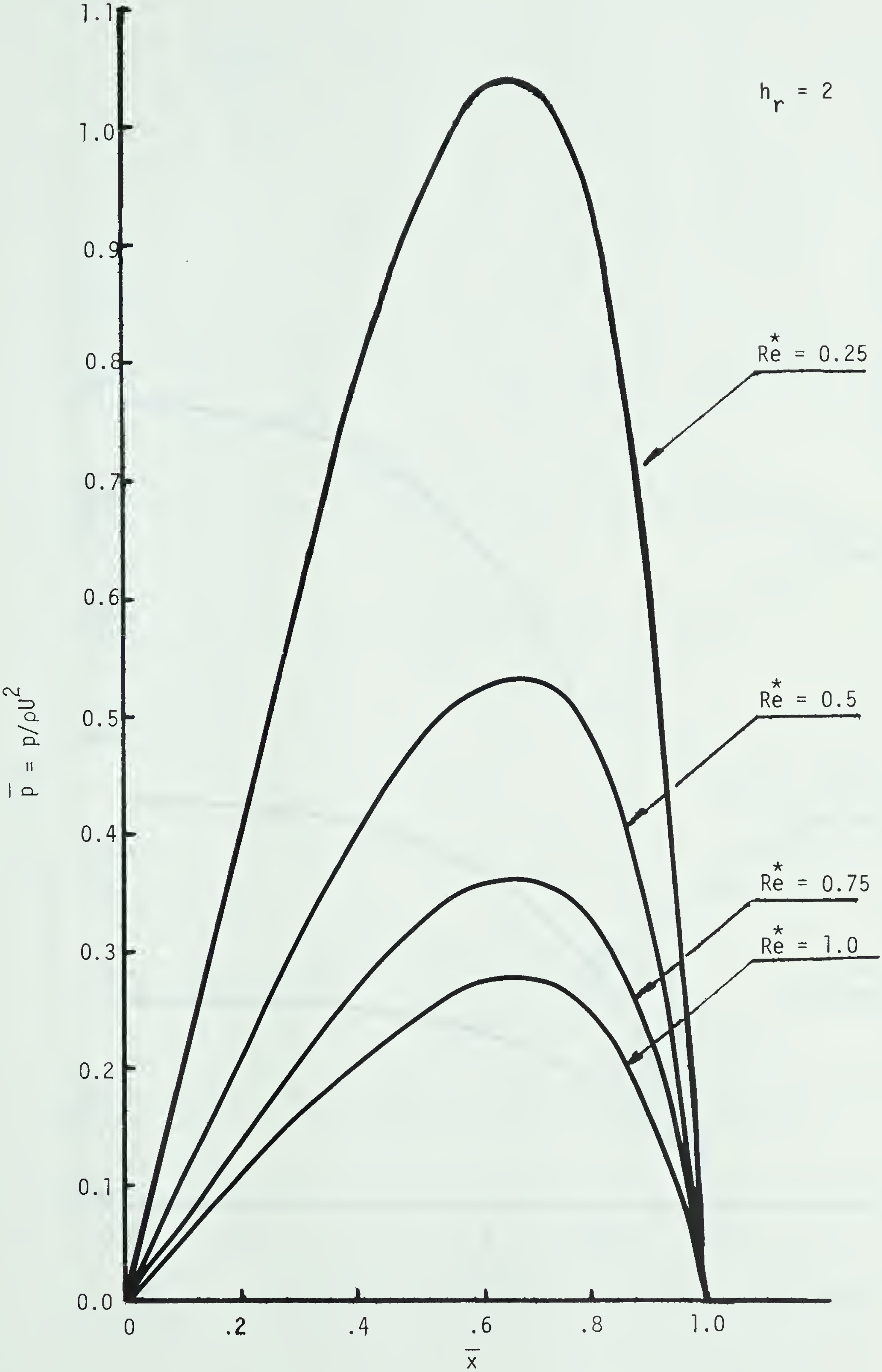

FIG. 4.3 PRESSURE DISTRIBUTION

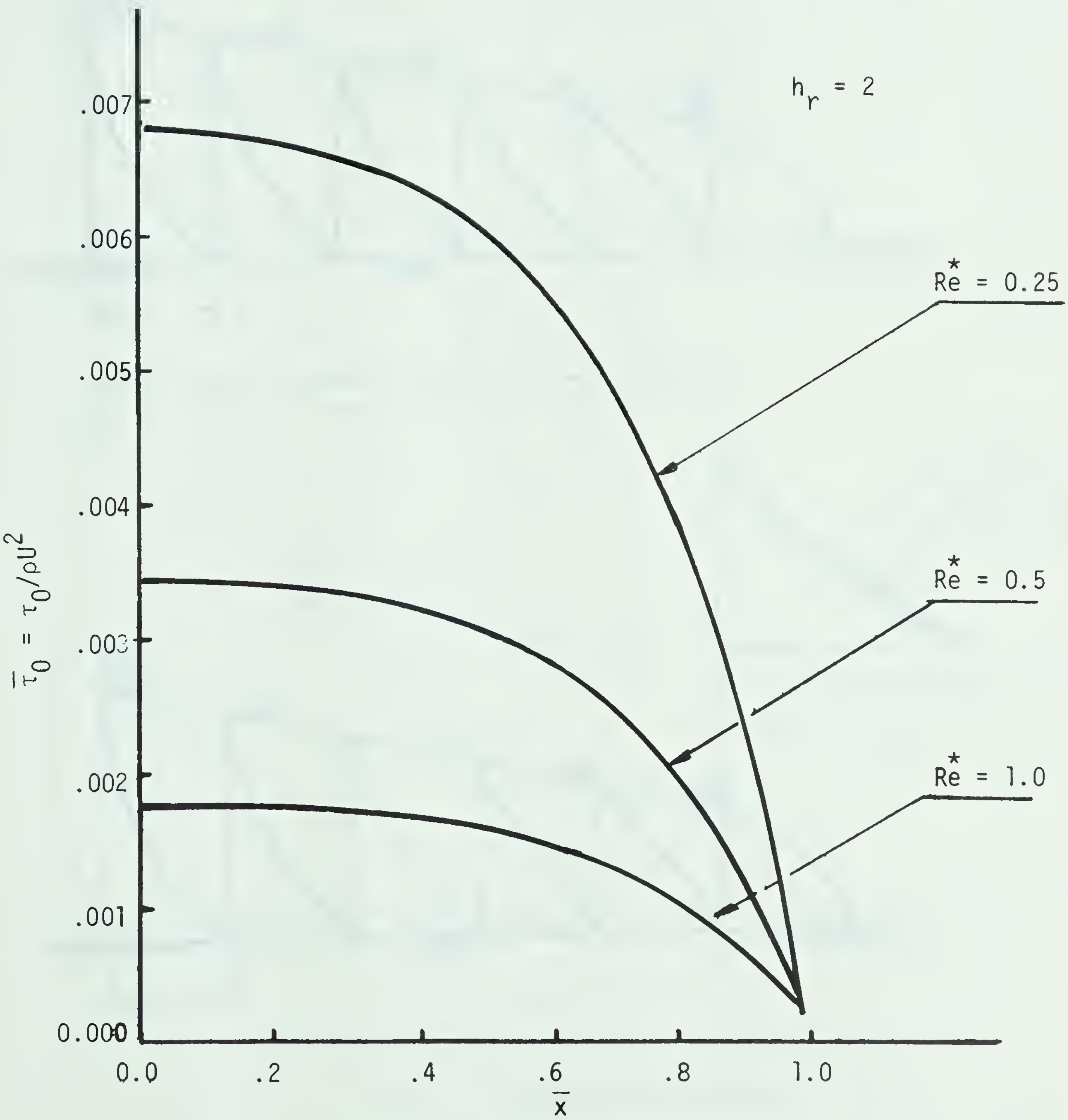

FIG. 4.4 SHEAR STRESS DISTRIBUTION AT THE MOVING COMPONENT

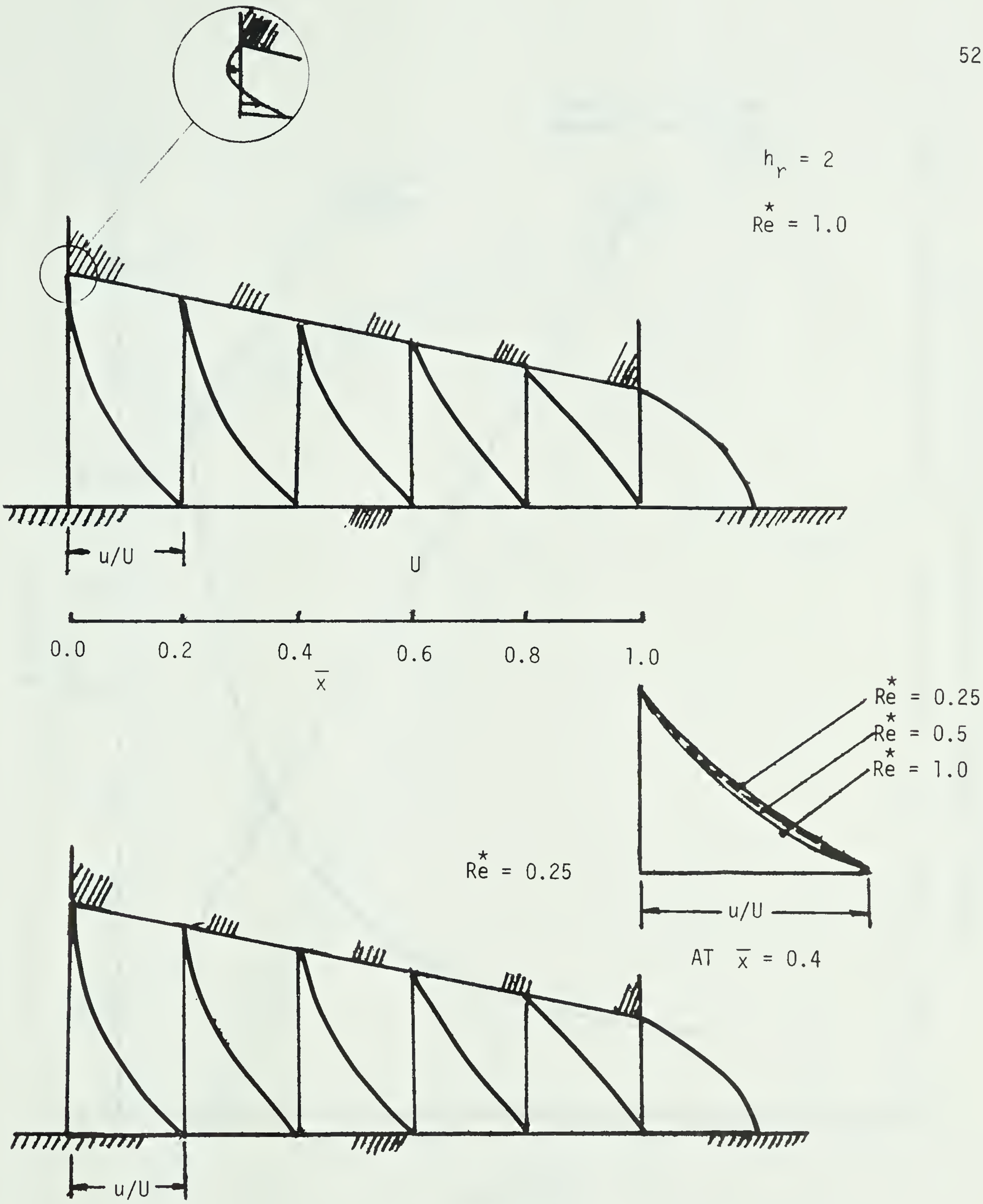

FIG. 4.5 VELOCITY DISTRIBUTION

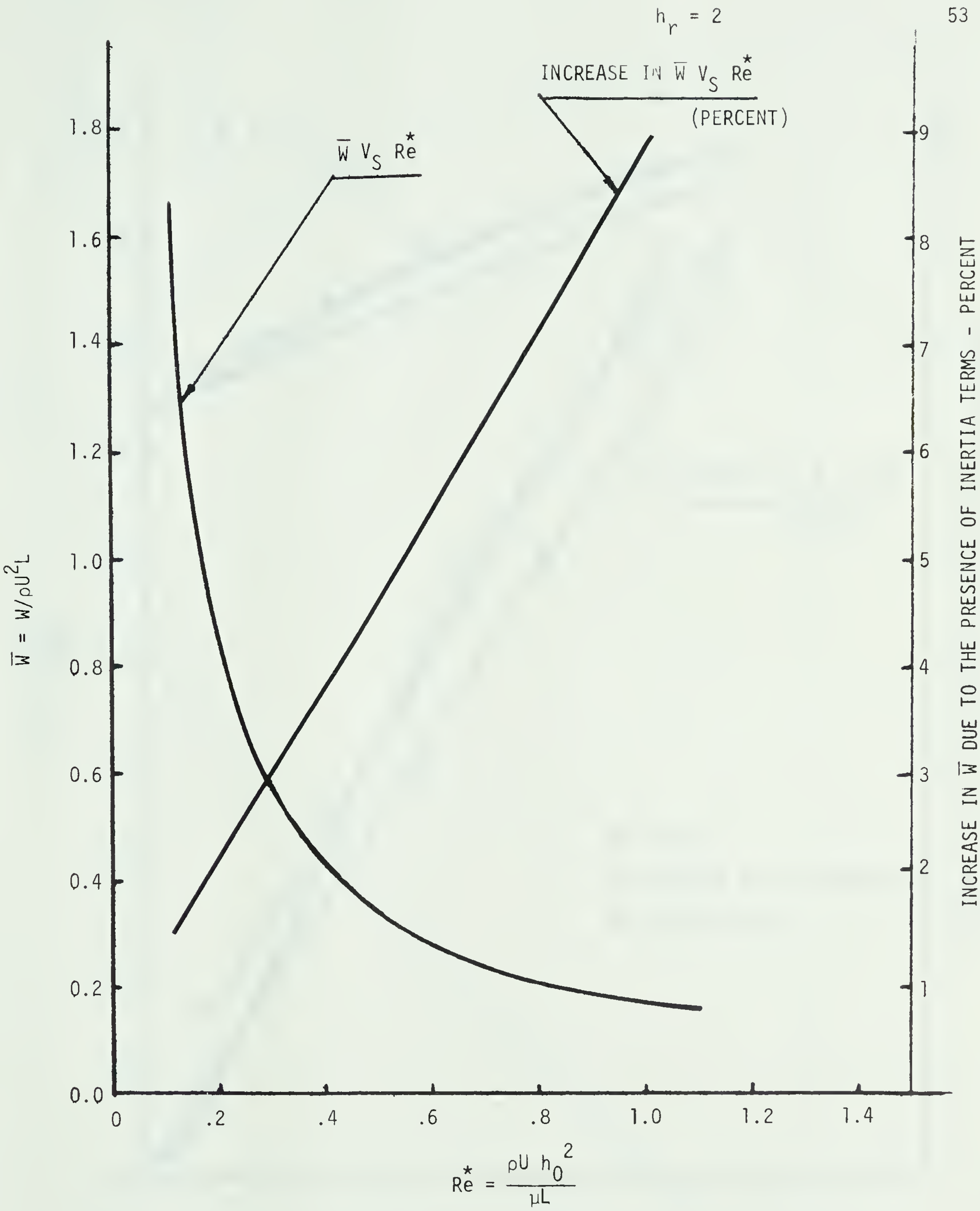

FIG. 4.6 LOAD CAPACITY

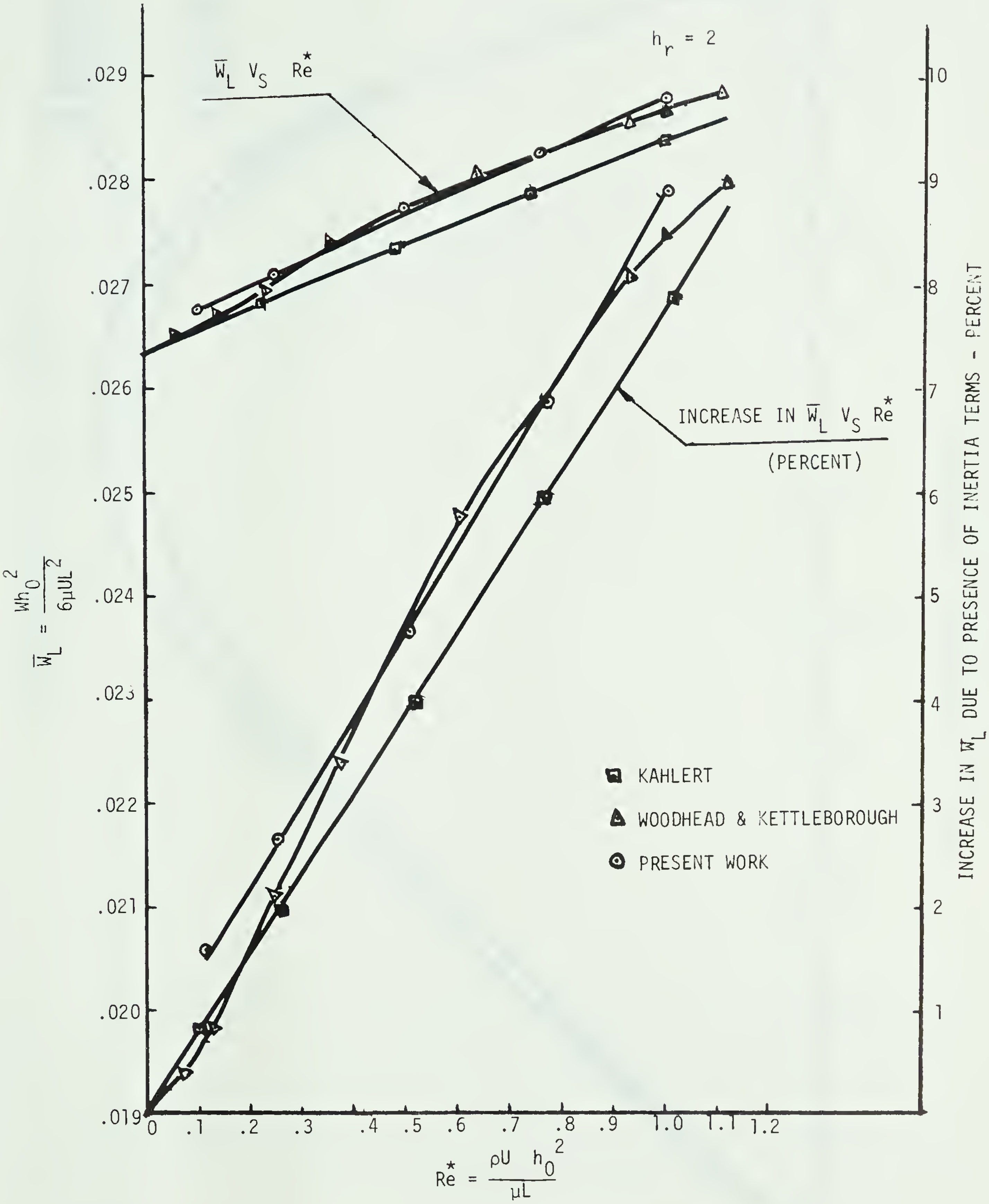

FIG. 4.7 LOAD CAPACITY

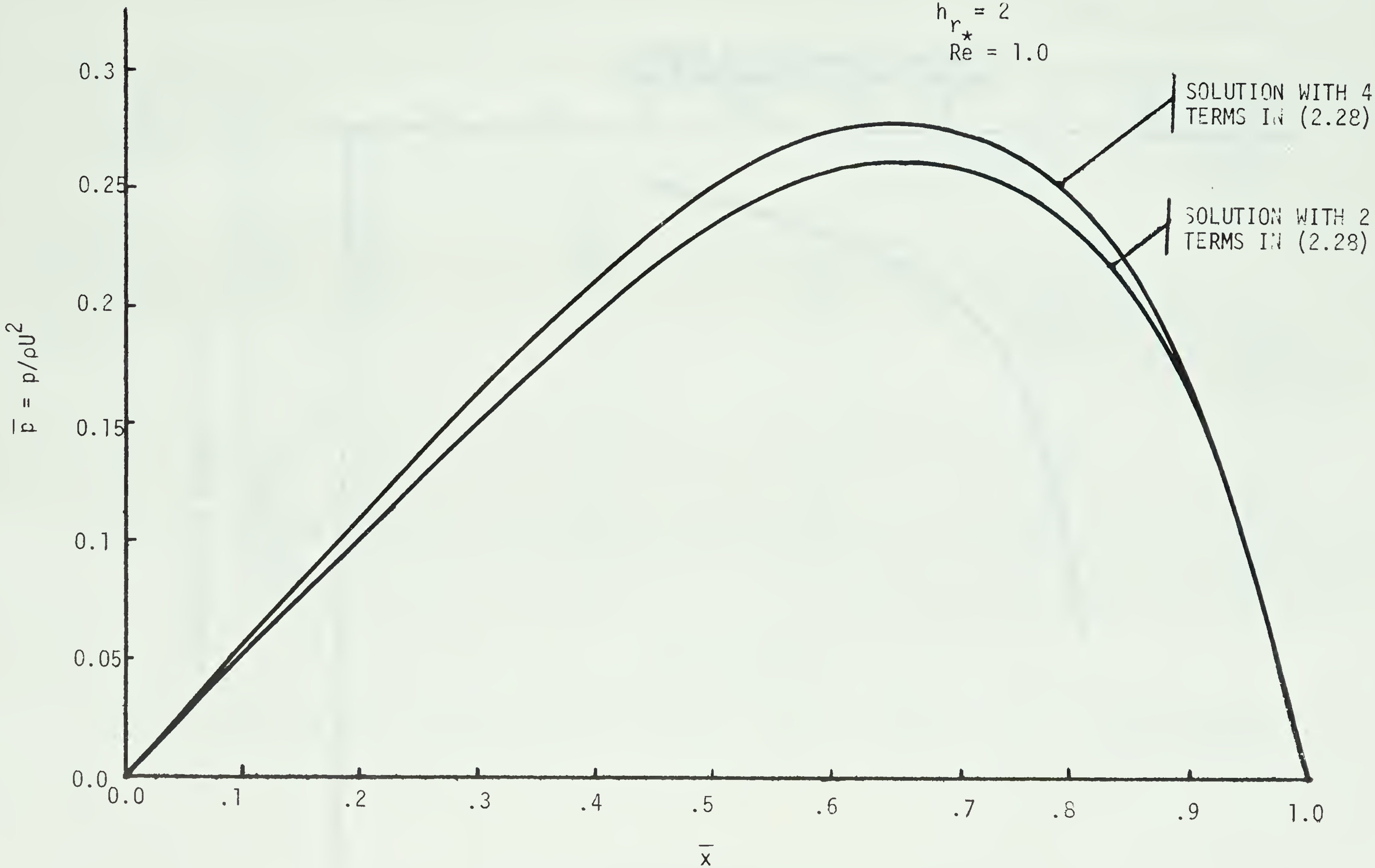

FIG. 4.8 PRESSURE DISTRIBUTION

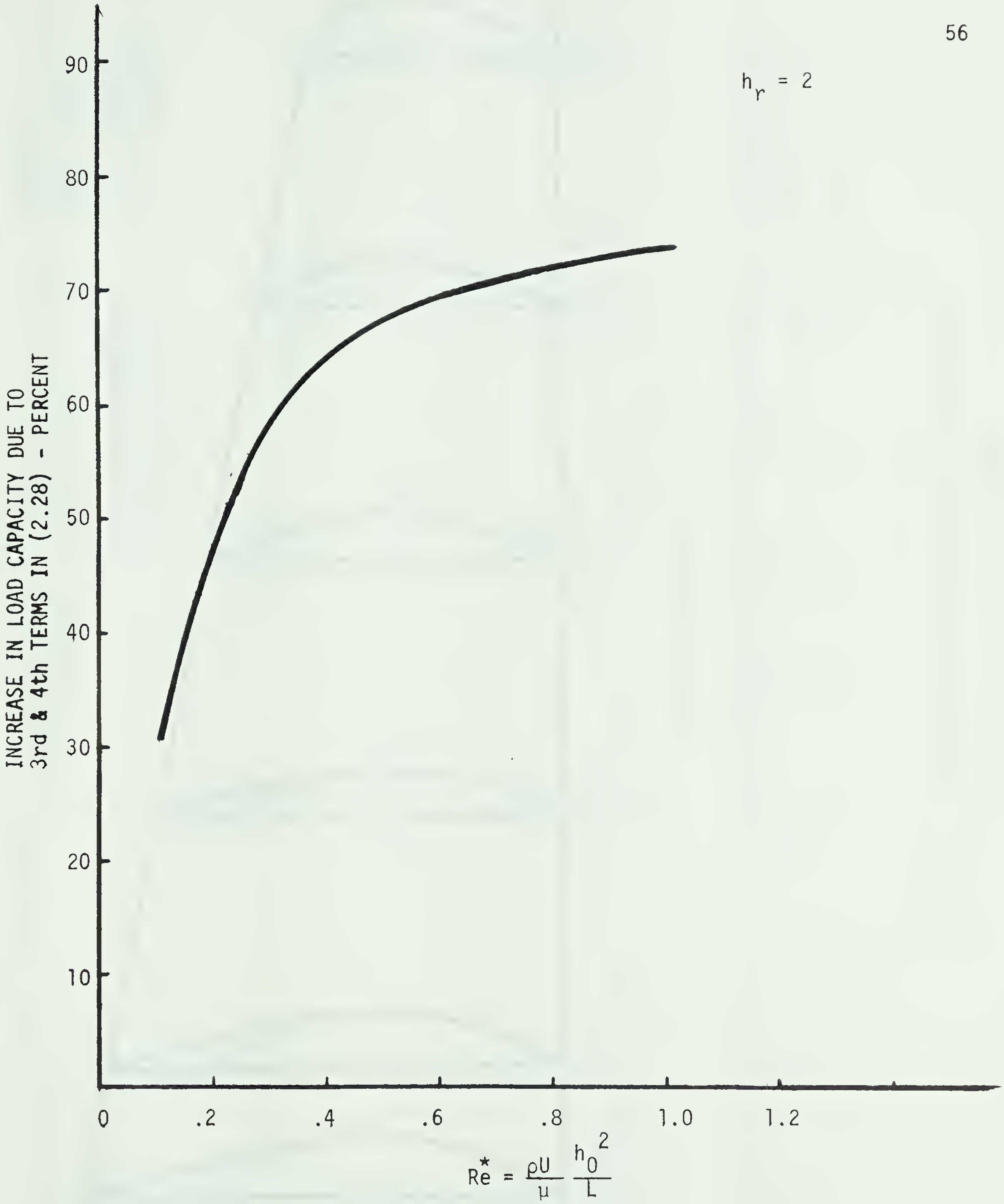

FIG. 4.9 INCREASE IN LOAD CAPACITY

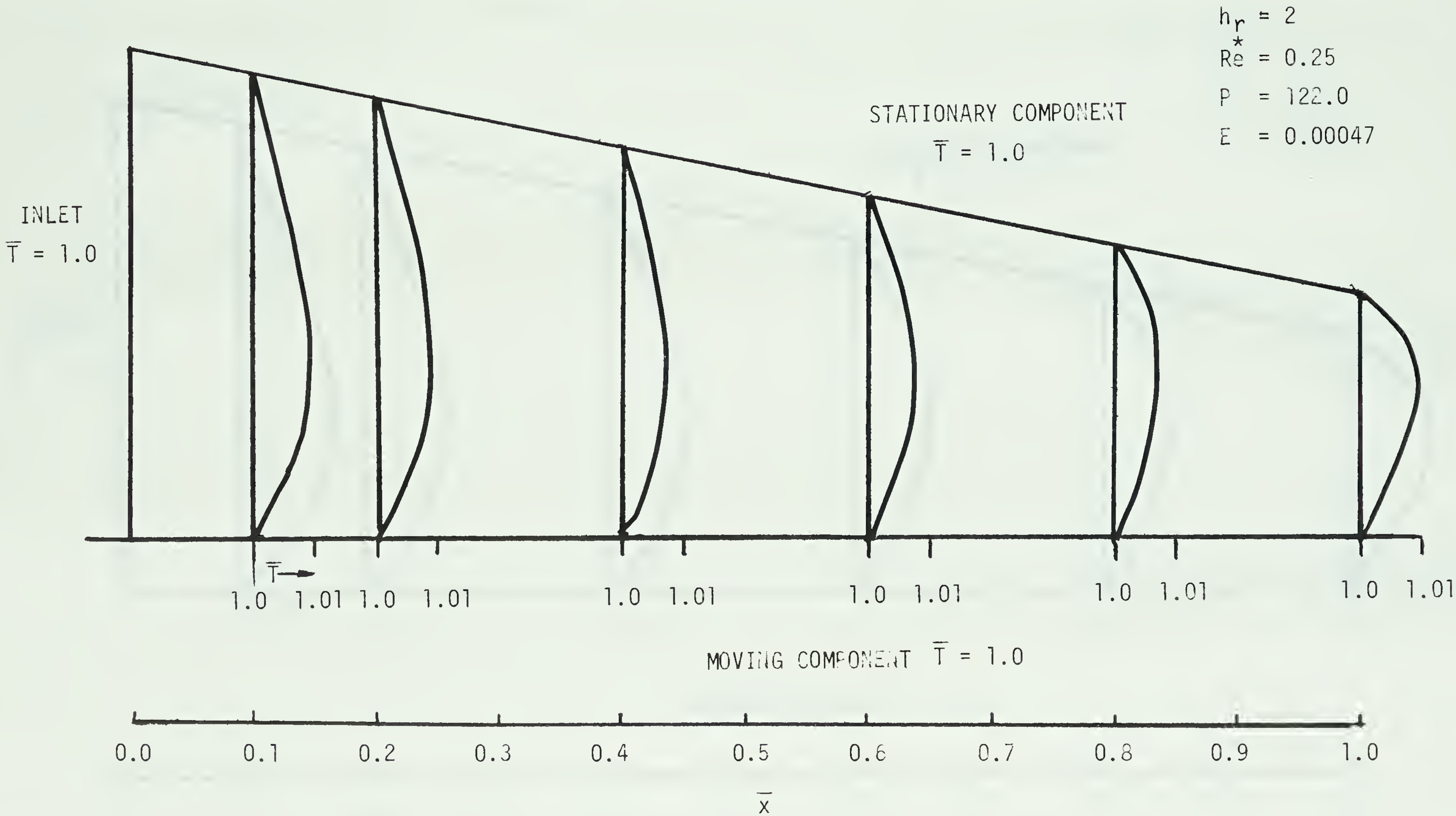

FIG. 4.10 TEMPERATURE DISTRIBUTION IN FLUID FILM

(WITHOUT CONVECTIVE TERMS)

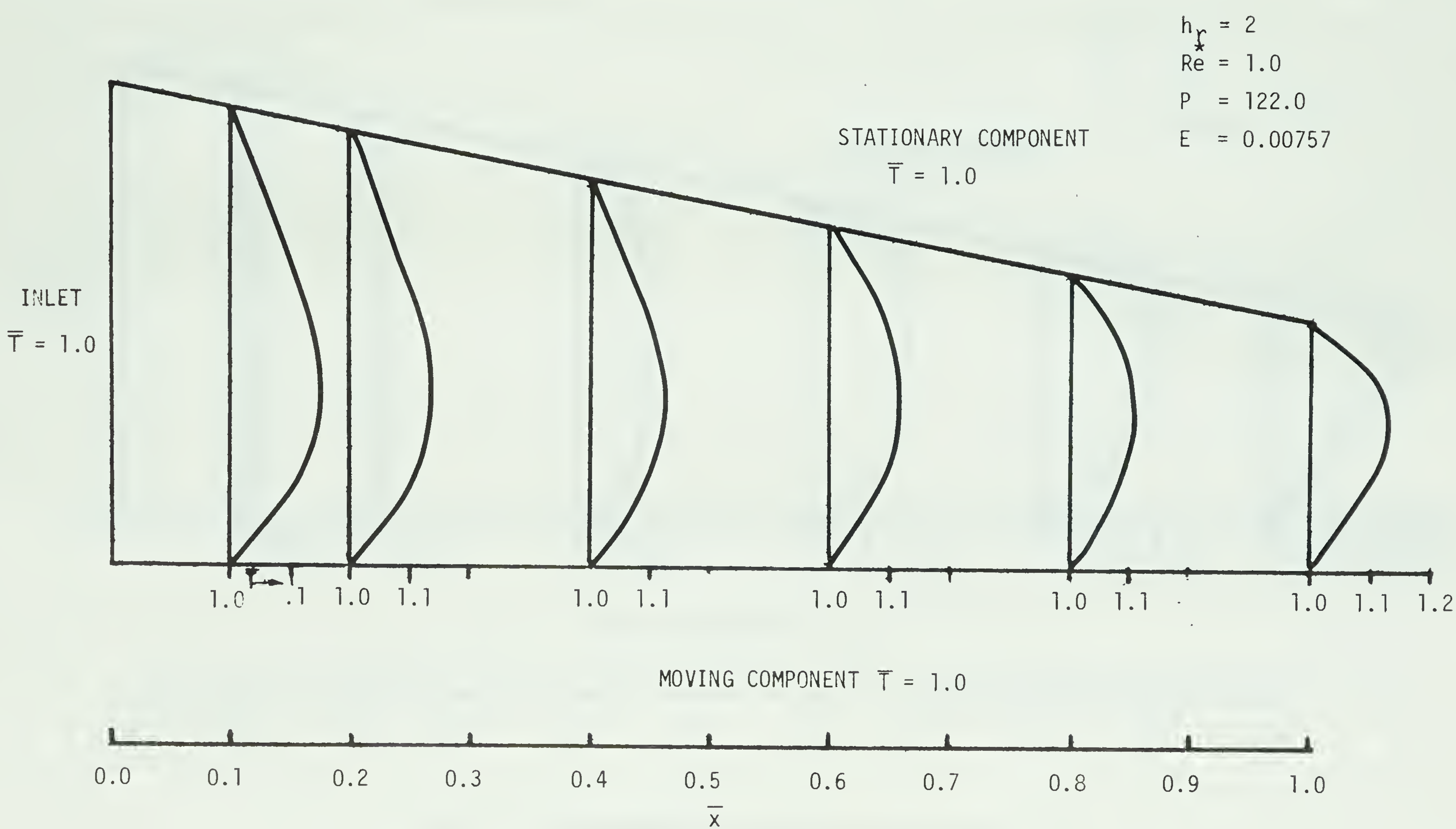

FIG. 4.11 TEMPERATURE DISTRIBUTION IN FLUID FILM

(WITHOUT CONVECTIVE TERMS)

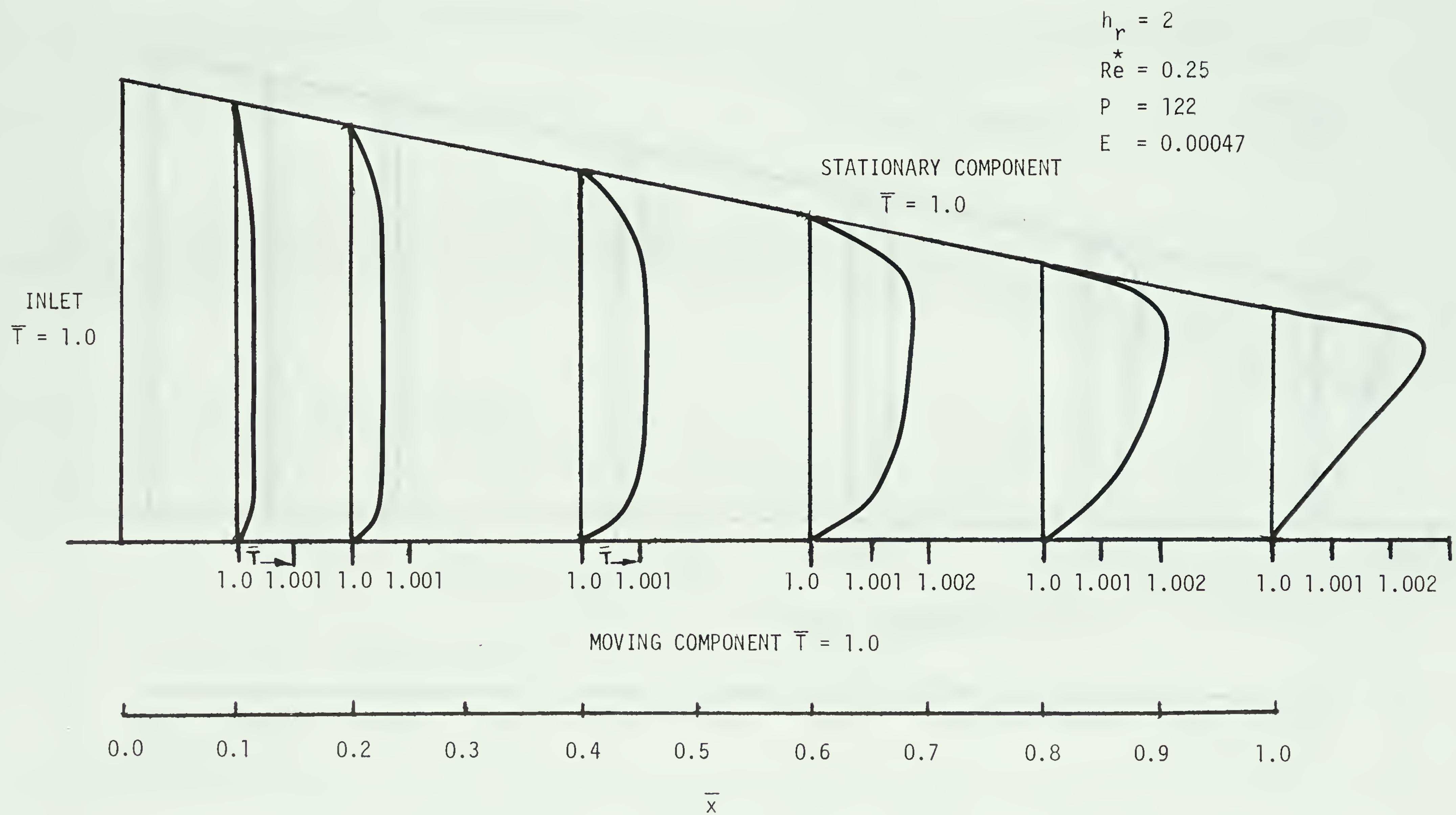

FIG. 4.12 TEMPERATURE DISTRIBUTION IN FLUID FILM

(WITH CONVECTIVE TERMS)

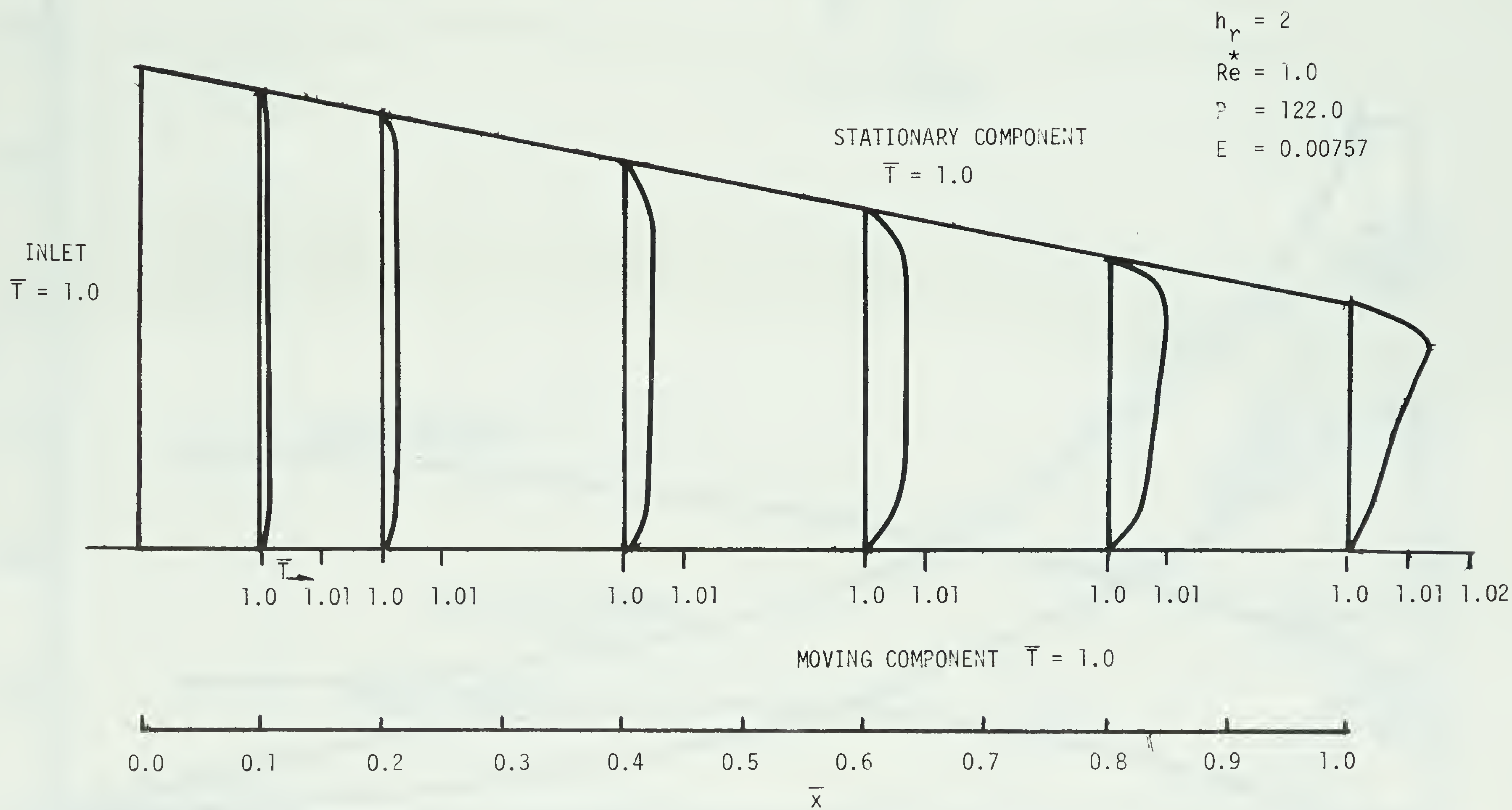

FIG. 4.13 TEMPERATURE DISTRIBUTION IN FLUID FILM

(WITH CONVECTIVE TERMS)

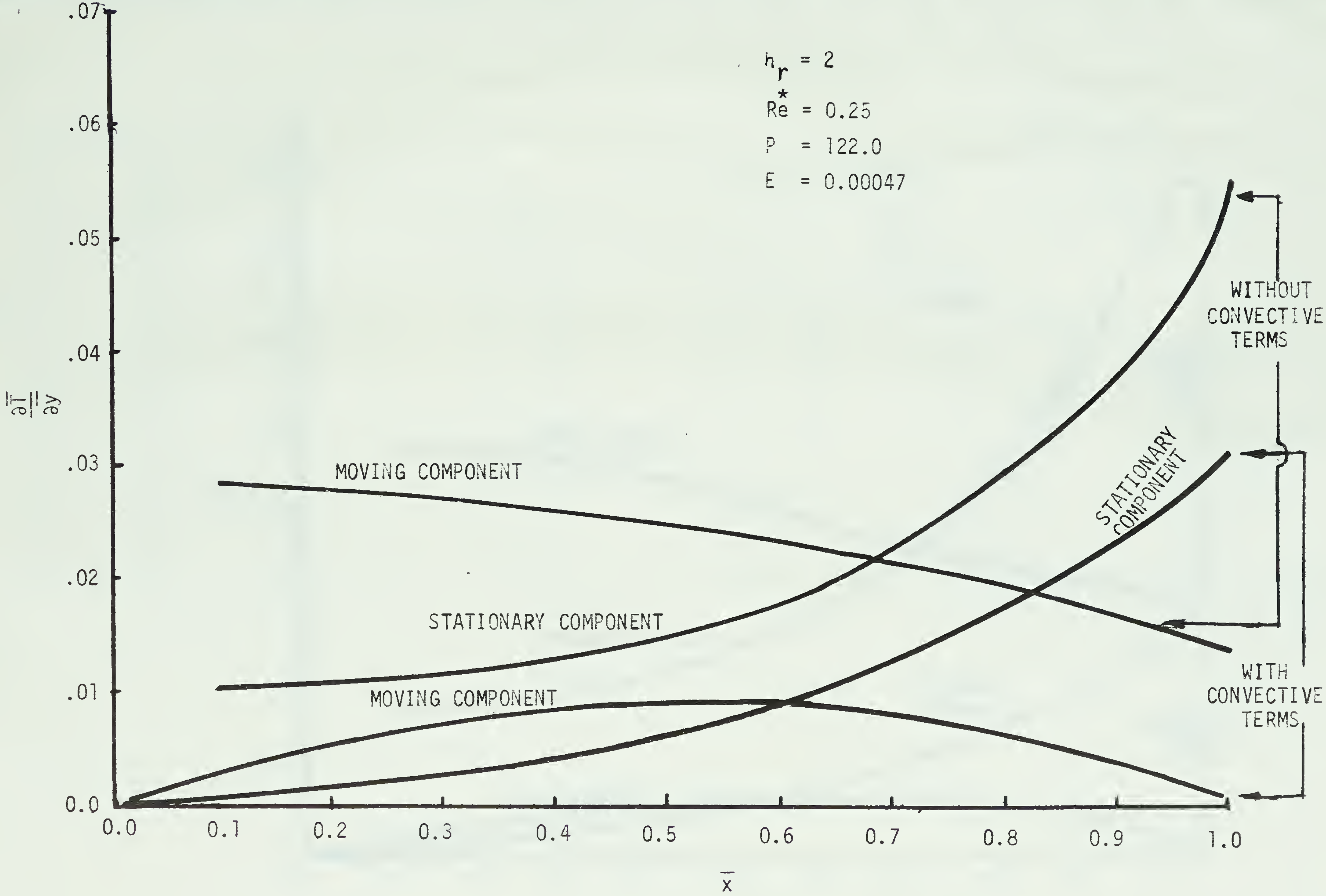

FIG. 4.14 TEMPERATURE GRADIENT ON MOVING AND STATIONARY COMPONENTS

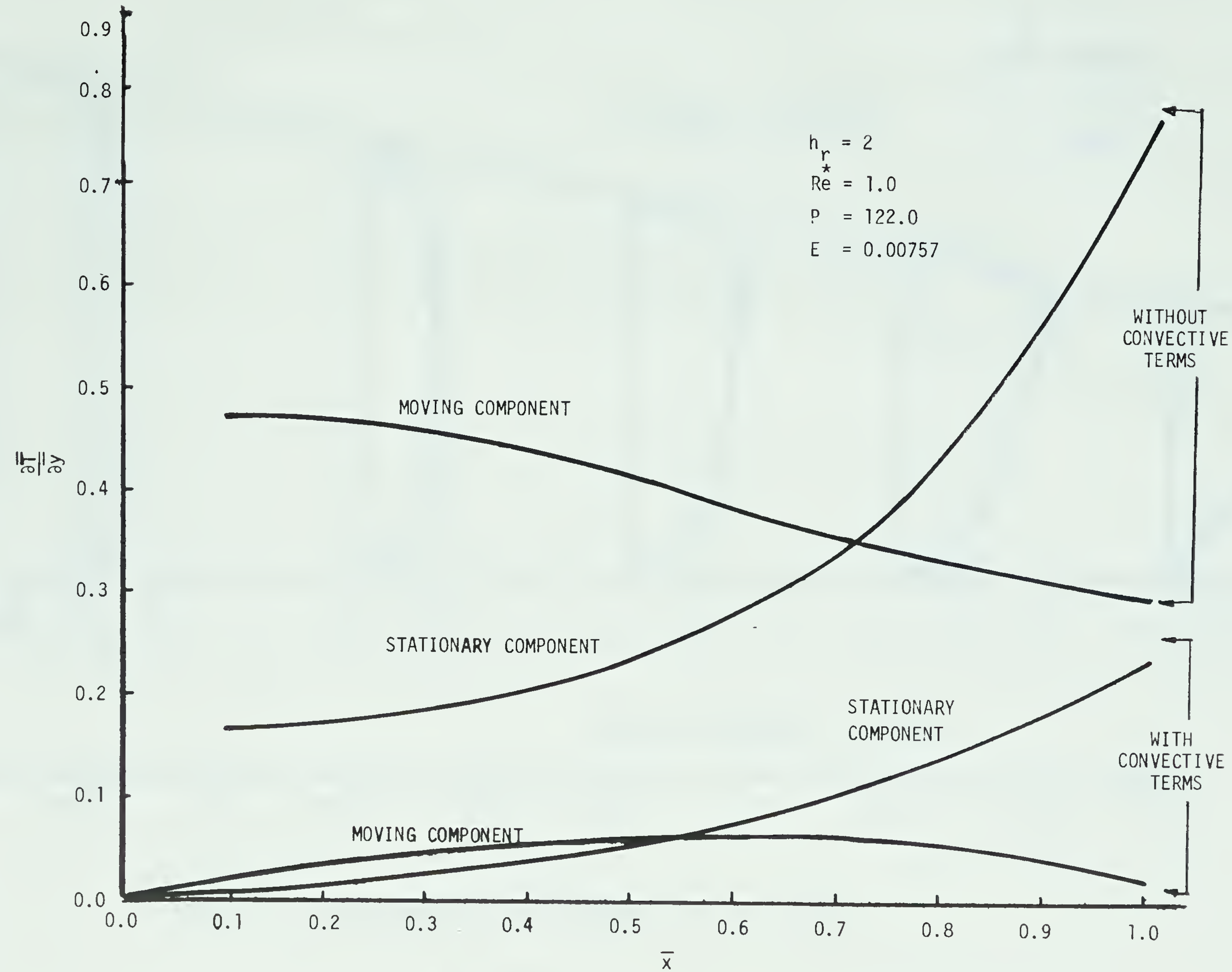

FIG. 4.15 TEMPERATURE GRADIENT ON MOVING AND STATIONARY COMPONENT

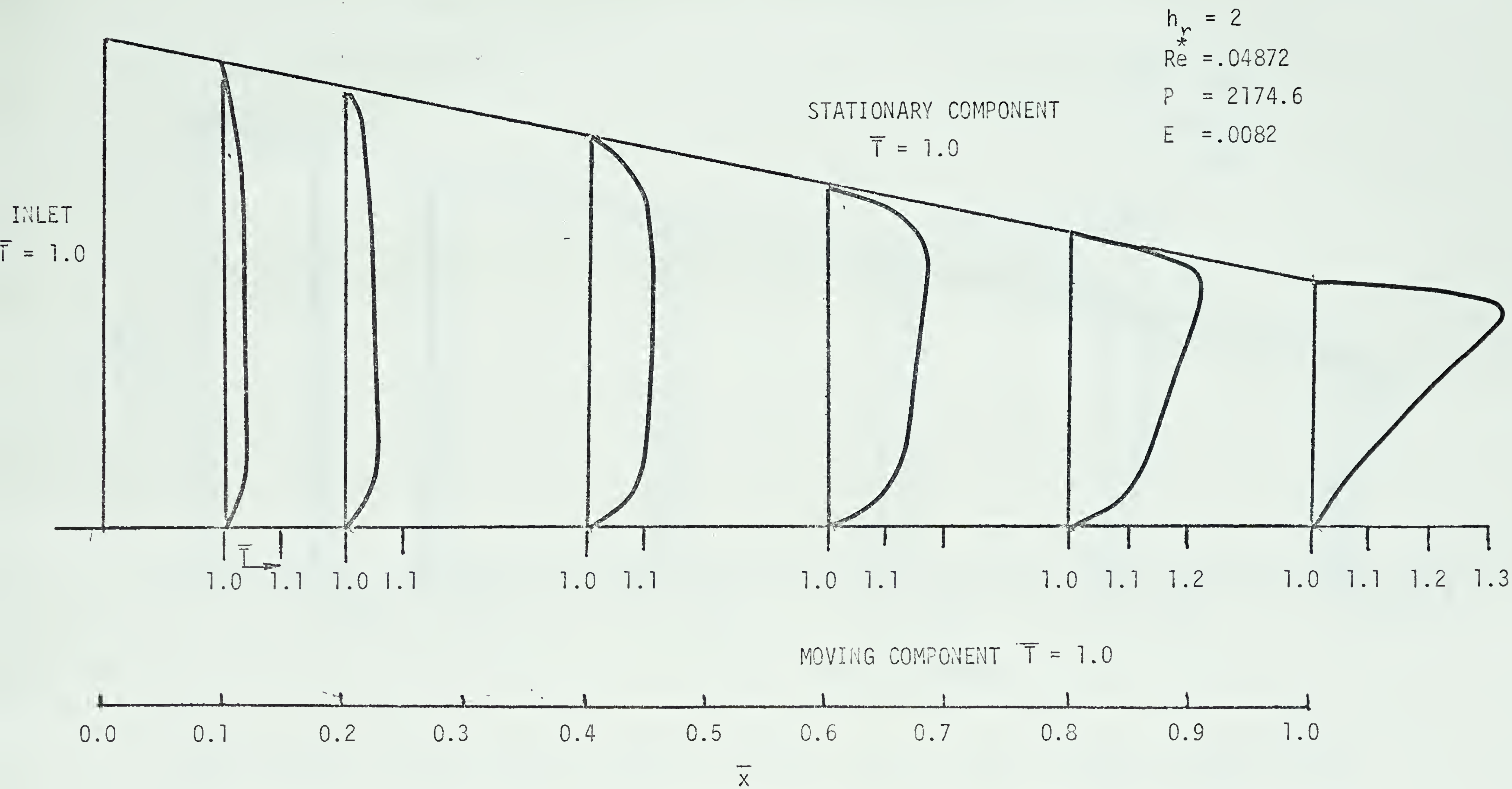

FIG. 4.16 TEMPERATURE DISTRIBUTION IN FLUID FILM

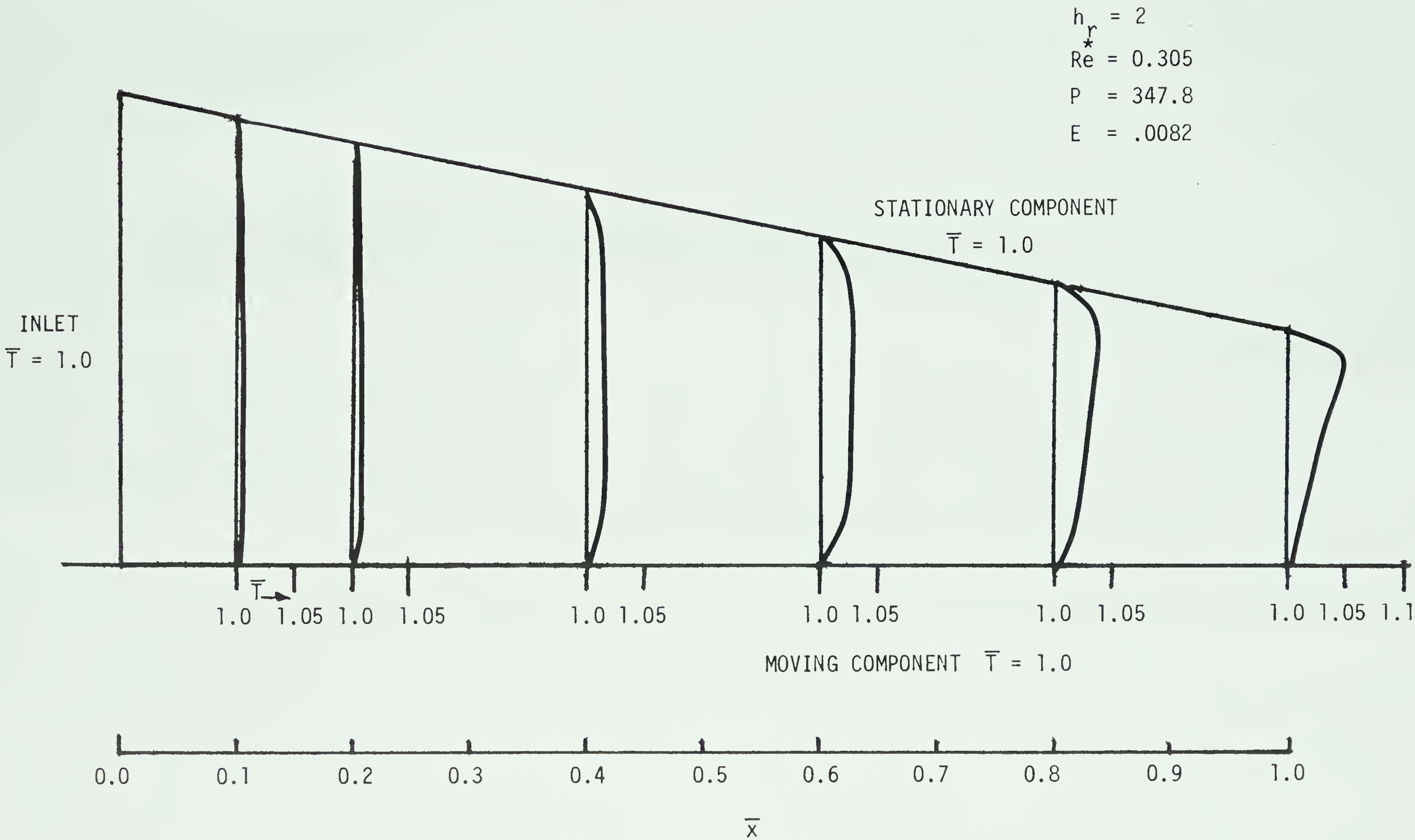

FIG. 4.17 TEMPERATURE DISTRIBUTION IN FLUID FILM